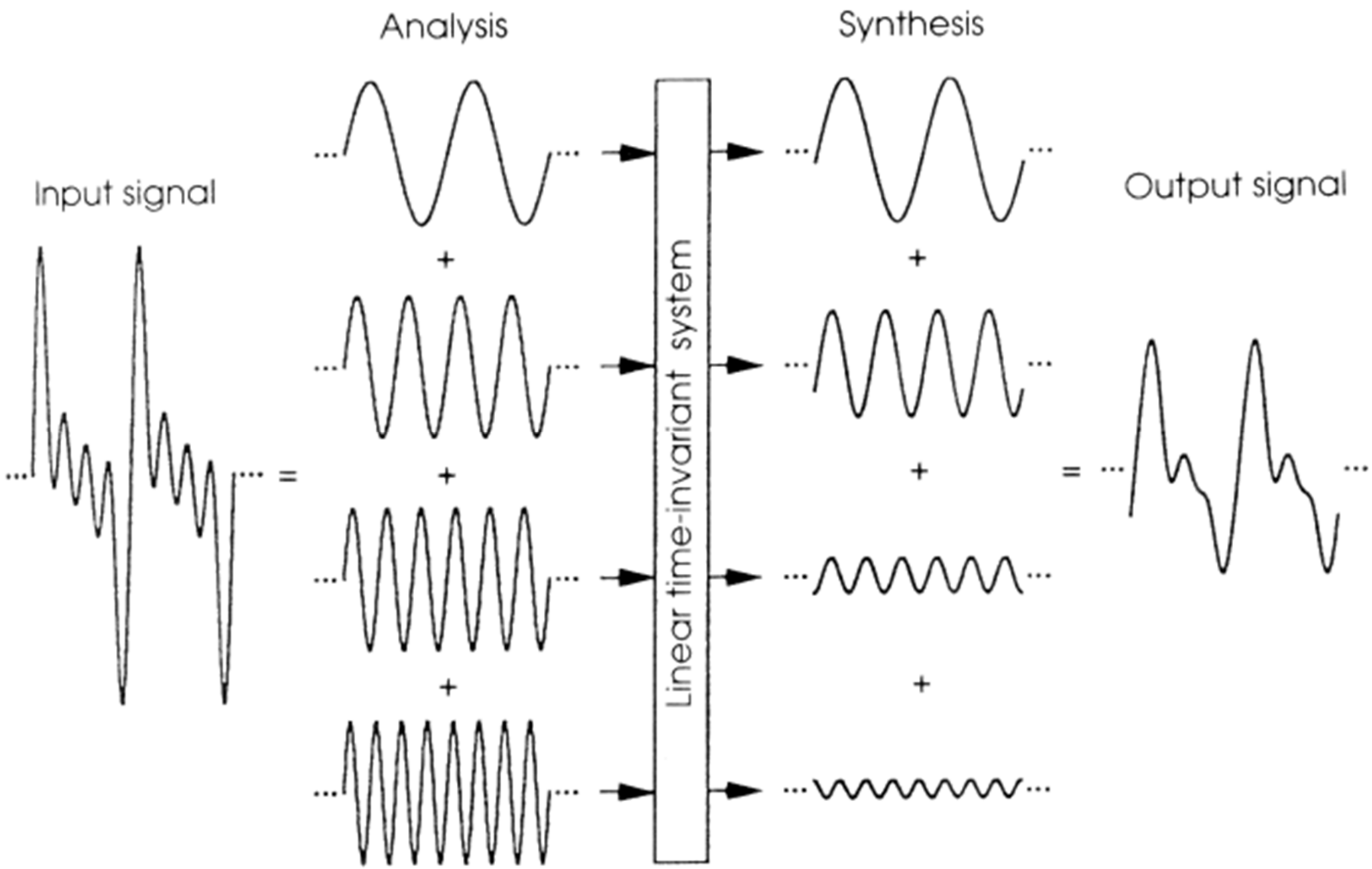
Signals, systems, acoustics and the ear

Week 3

Frequency characterisations of systems & signals

The big idea

As long as we know what the system does to sinusoids...



... we can predict any output to any input.

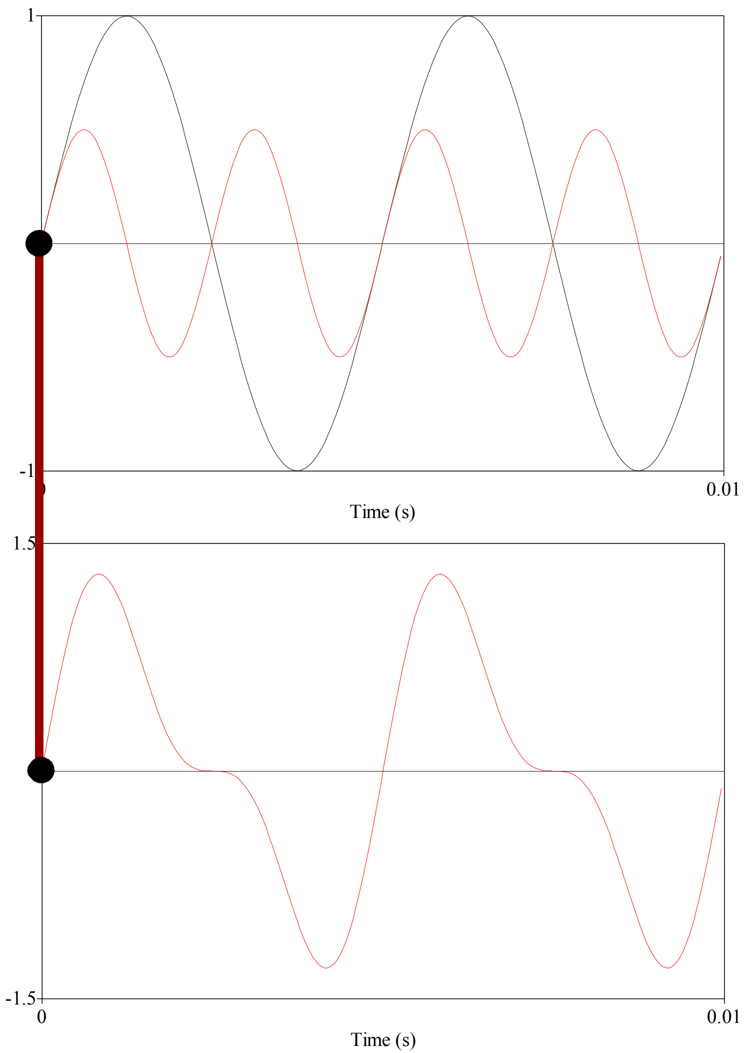
Representing signals as sums of sinusoids: Spectra

Synthesising waves

French mathematician
Jean Baptiste Joseph Fourier
1768-1830

Fourier Synthesis

we add up sinewaves by adding up the respective amplitude values of all sine waves for each point in time



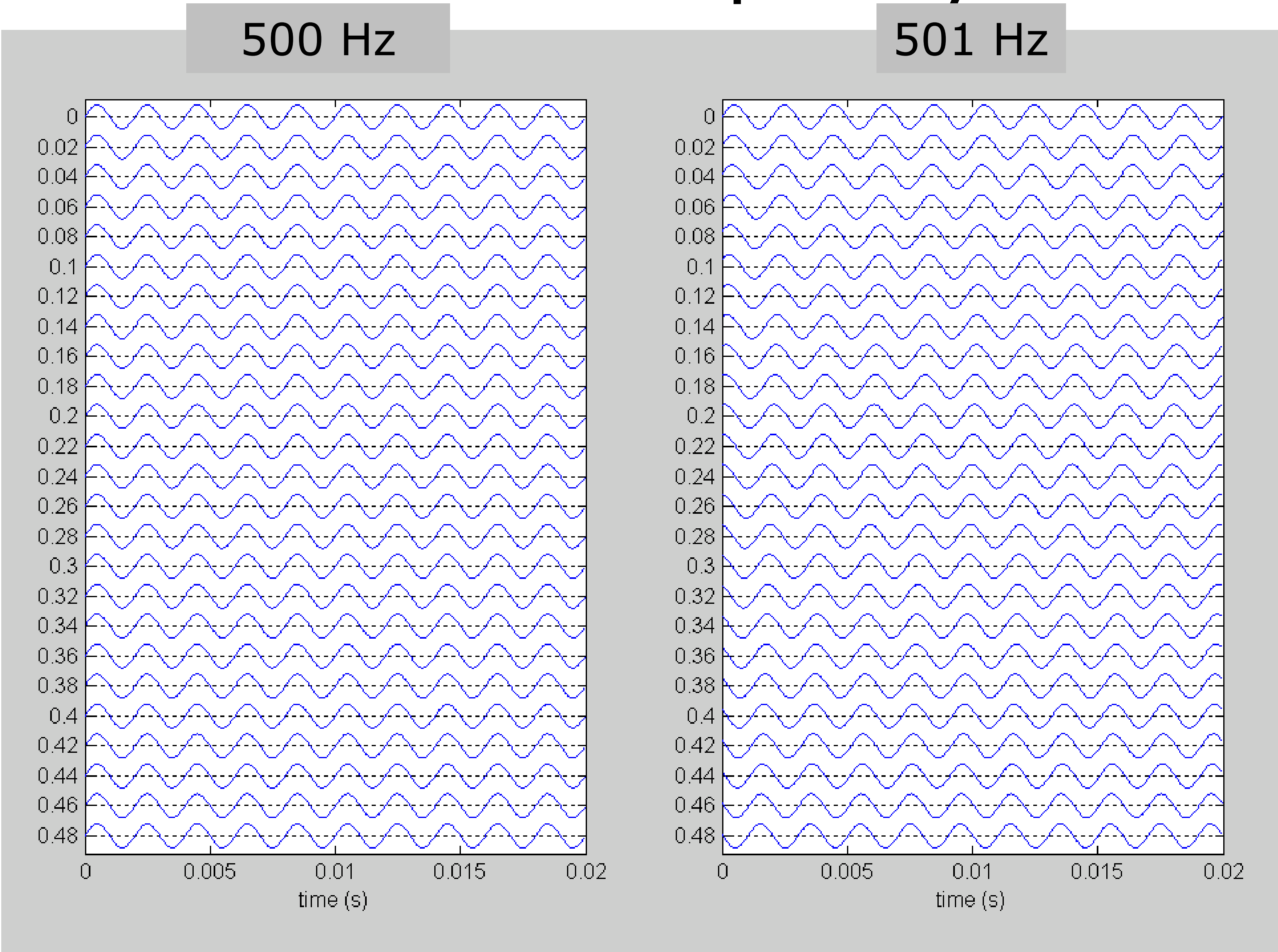
sinewave I: 200 Hz

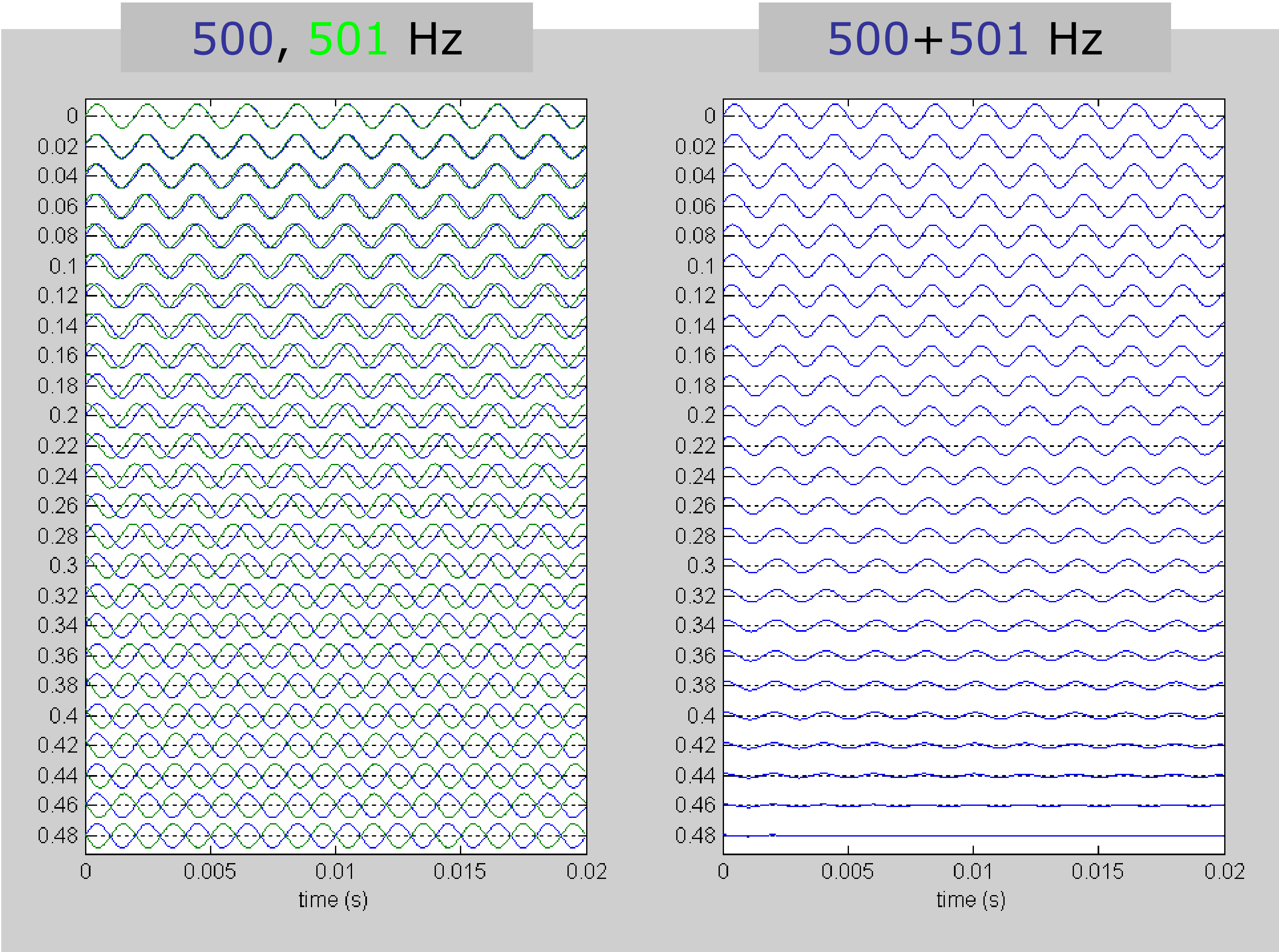
+

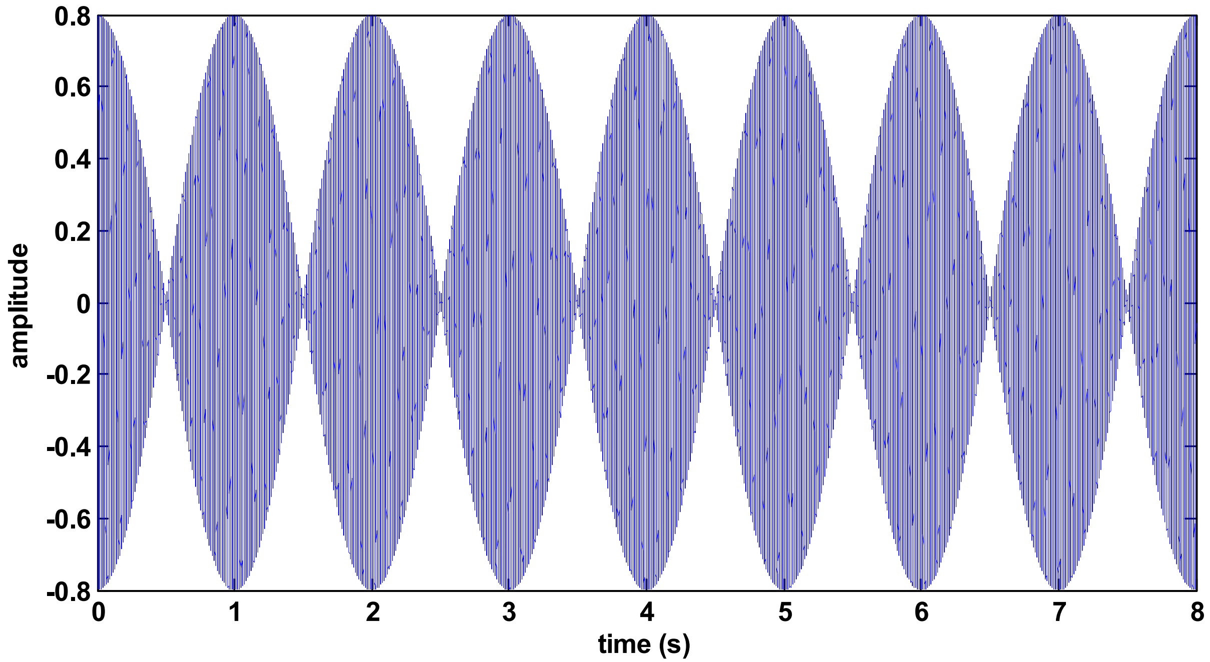
sinewave II: 400 Hz

this leads to a complex waveform consisting of a 200 and a 400 Hz sinusoid

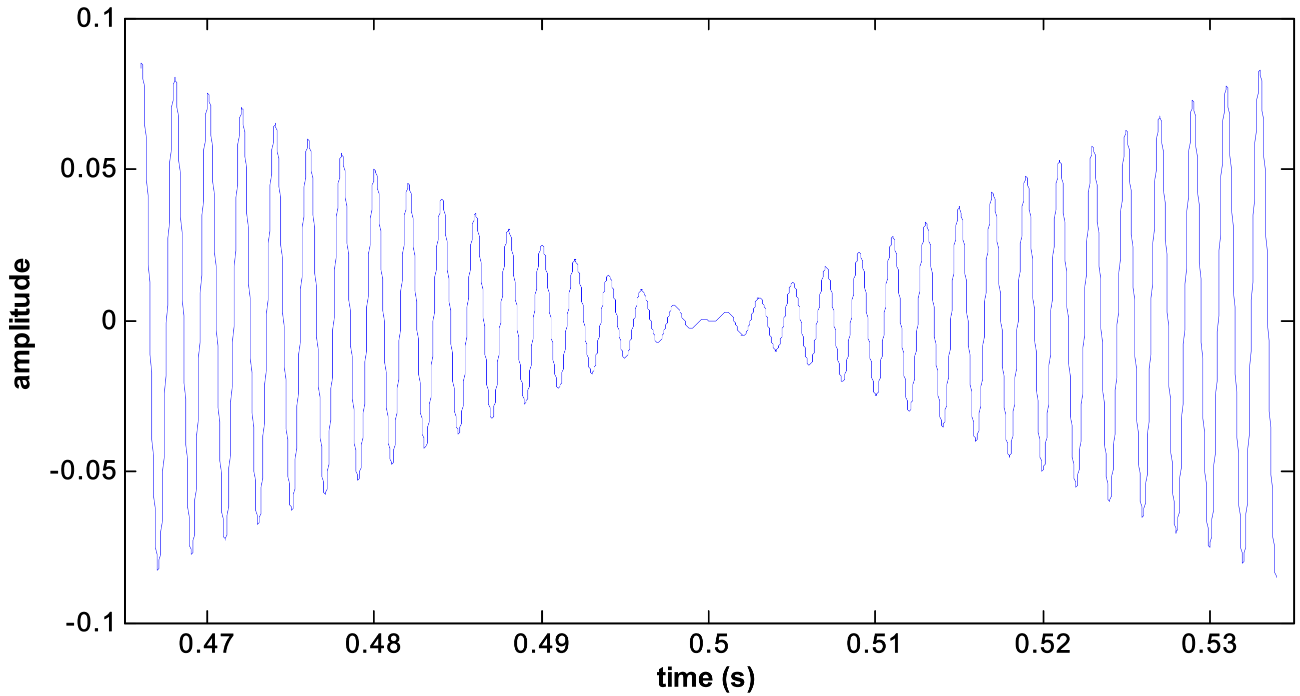
Beats: Add 2 sinewaves that are close in frequency




500, 501 Hz
500+501 Hz
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0.22
0.24
0.26
0.28
0.3
0.32
0.34
0.36
0.38
0.4
0.42
0.44
0.46
0.48
0
0.005
0.01
0.015
0.02
time (s)



amplitude
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
1
2
3
4
5
6
7
8
time (s)



amplitude
0.1
0.05
0
-0.05
-0.1
0.47
0.48
0.49
0.5
0.51
0.52
0.53
time (s)

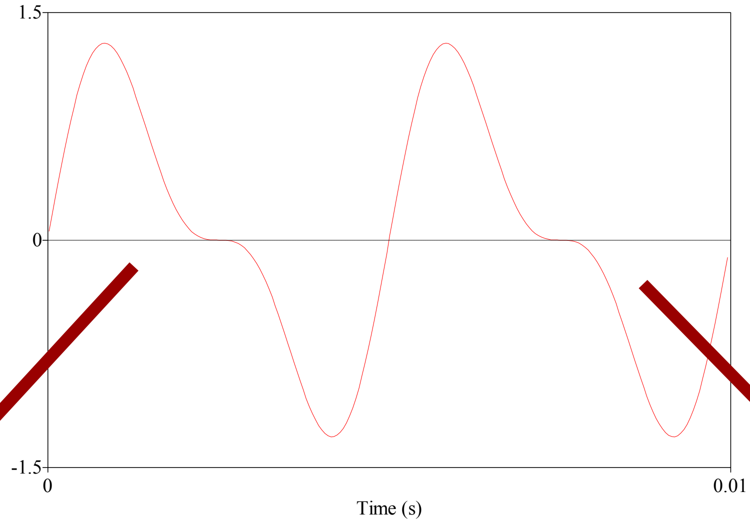
Fourier Analysis

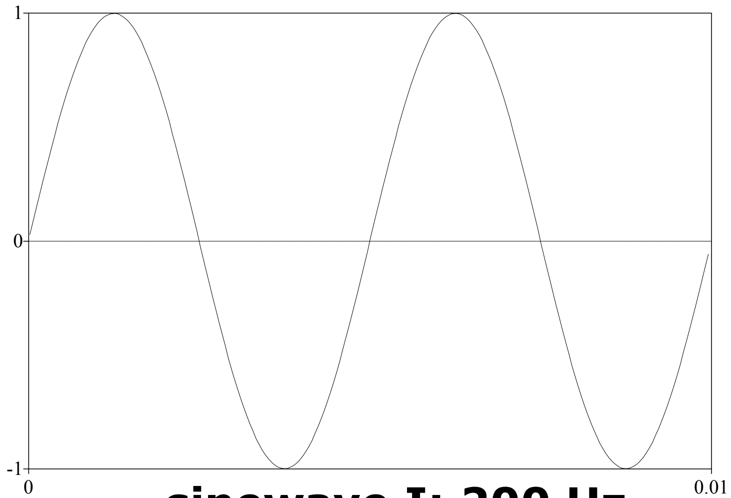
Fourier series analysis

(calculus based)

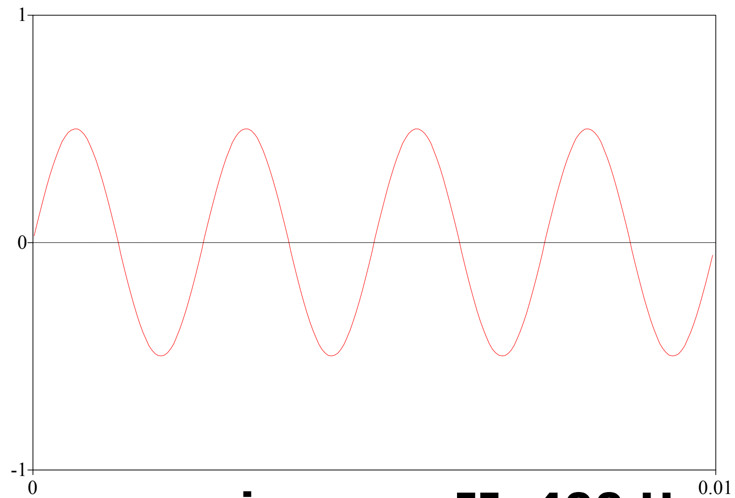
Suppose we are given a complex waveform:

The question is, which are the underlying sine waves?





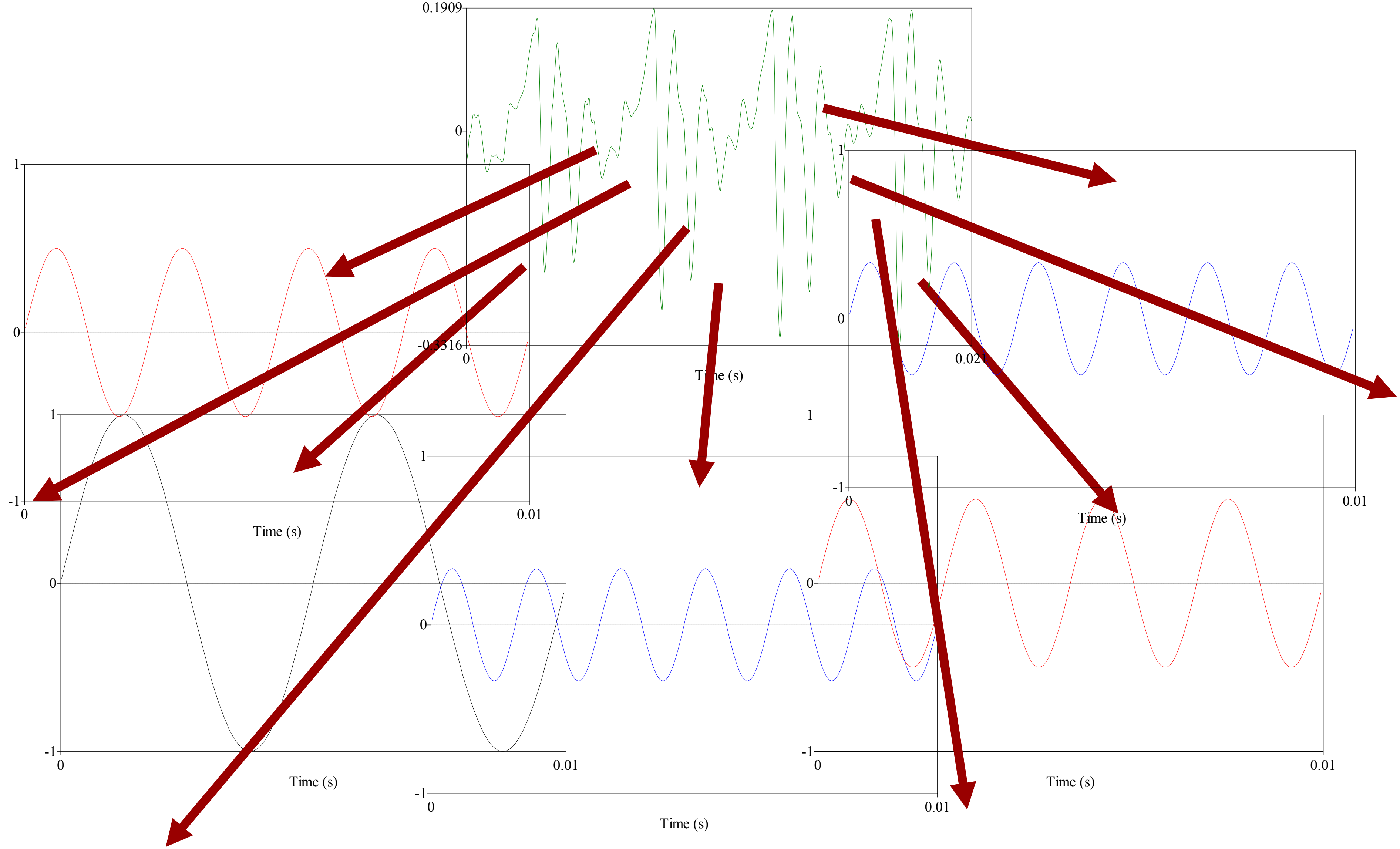
sinewave I: 200 Hz



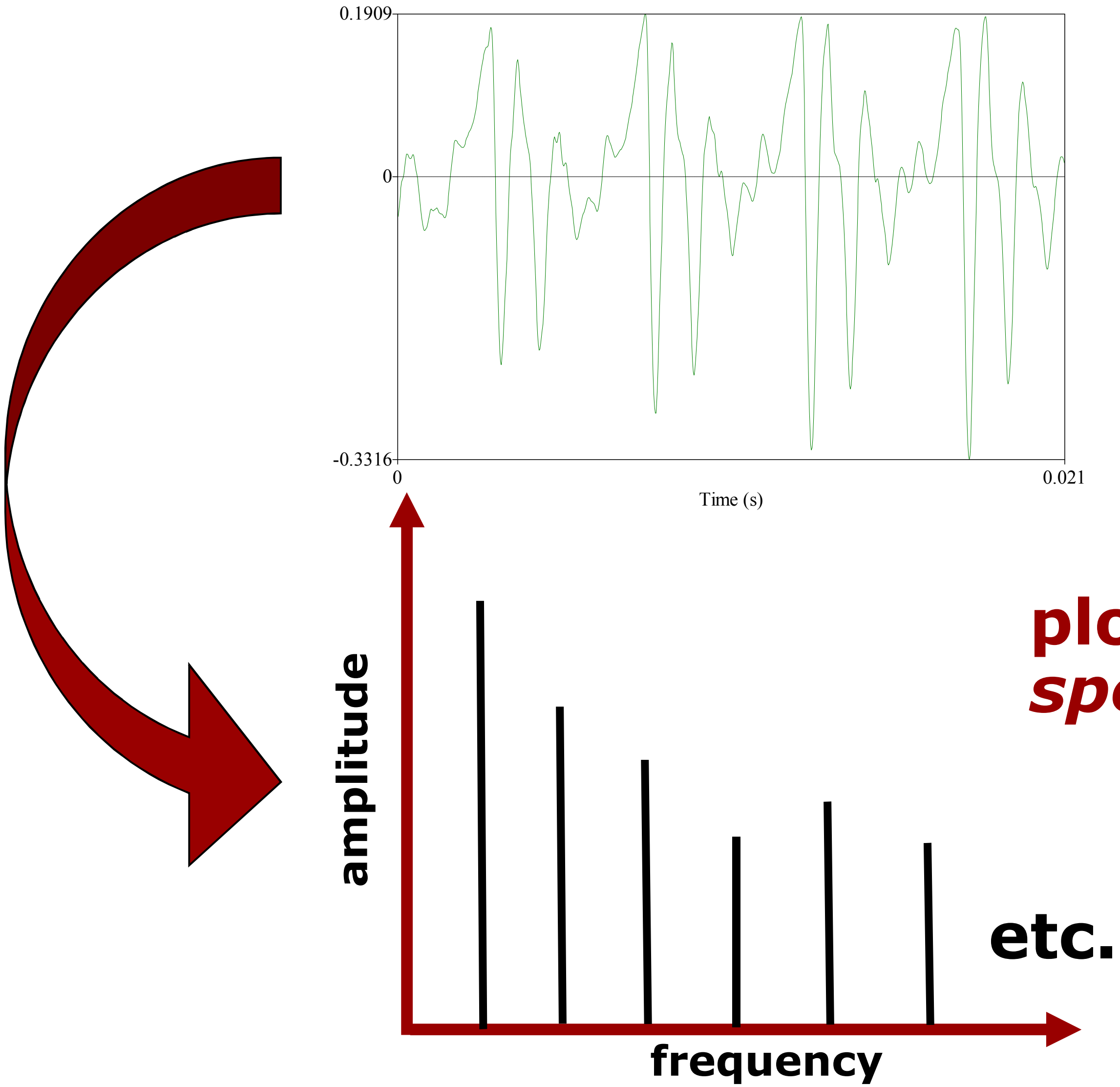
sinewave II: 400 Hz

Fourier Analysis

What if the complex wave is really complex?



Fourier Analysis



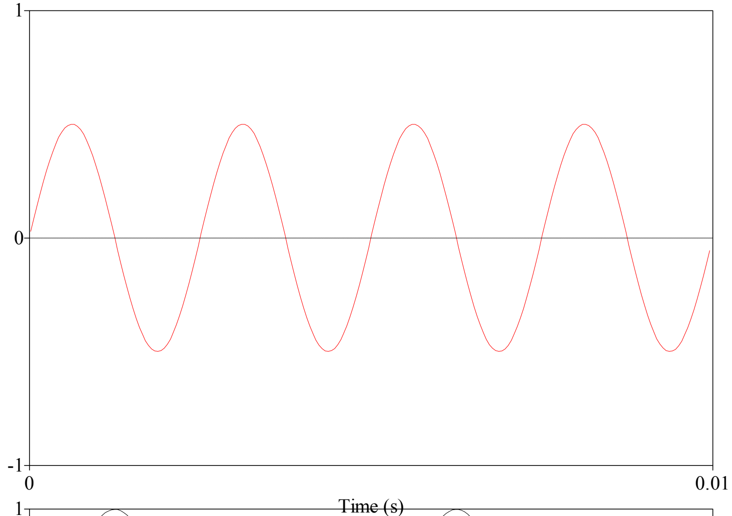
plotted as a *spectrum*

How to determine a spectrum

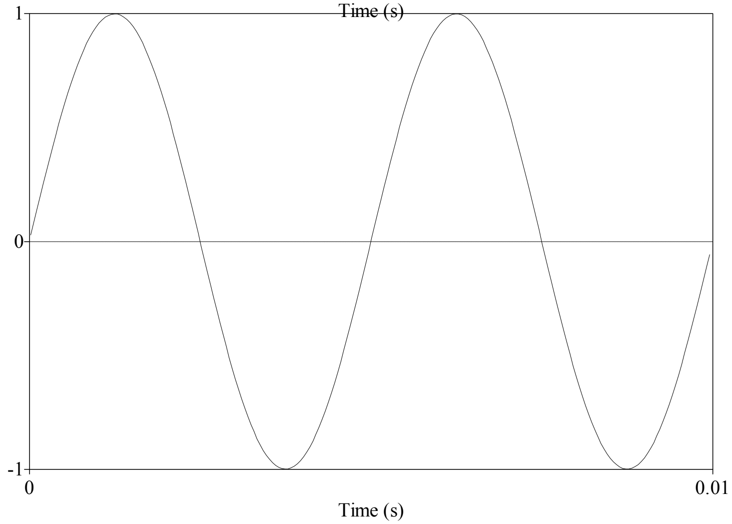
- Easy to see how to *synthesise*
  - spectrum $\rightarrow$ waveform
- But how do we analyse?
  - waveform $\rightarrow$ spectrum
- A special case: periodic complex waves
  - All component sine waves must be ***harmonically*** related
  - Their frequencies must be integer (whole-number) multiples of the repetition frequency of the complex waveform

Adding more than two sinusoids: component sine waves

400 Hz
½ V

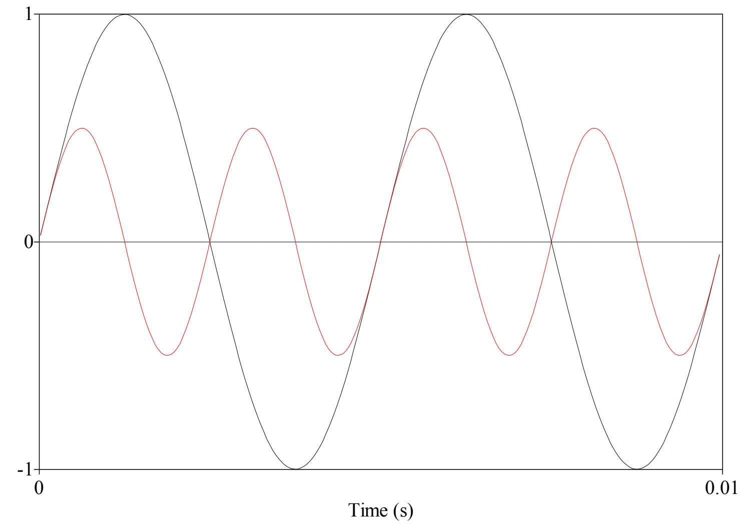


200 Hz
1 V

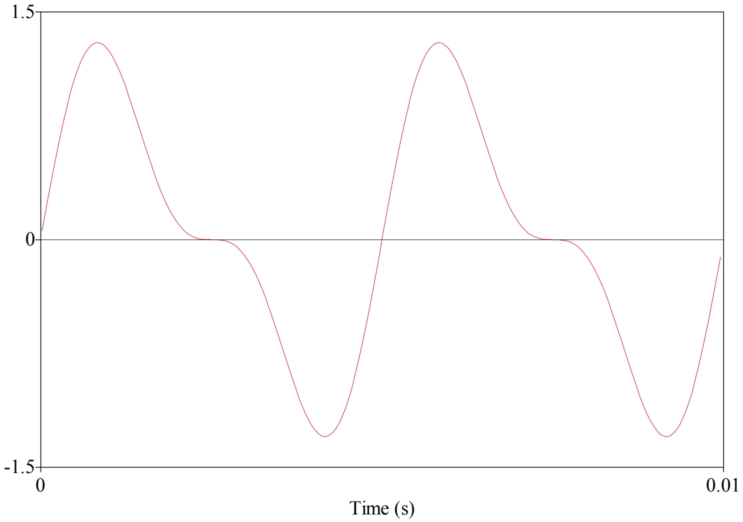


Adding Waveforms

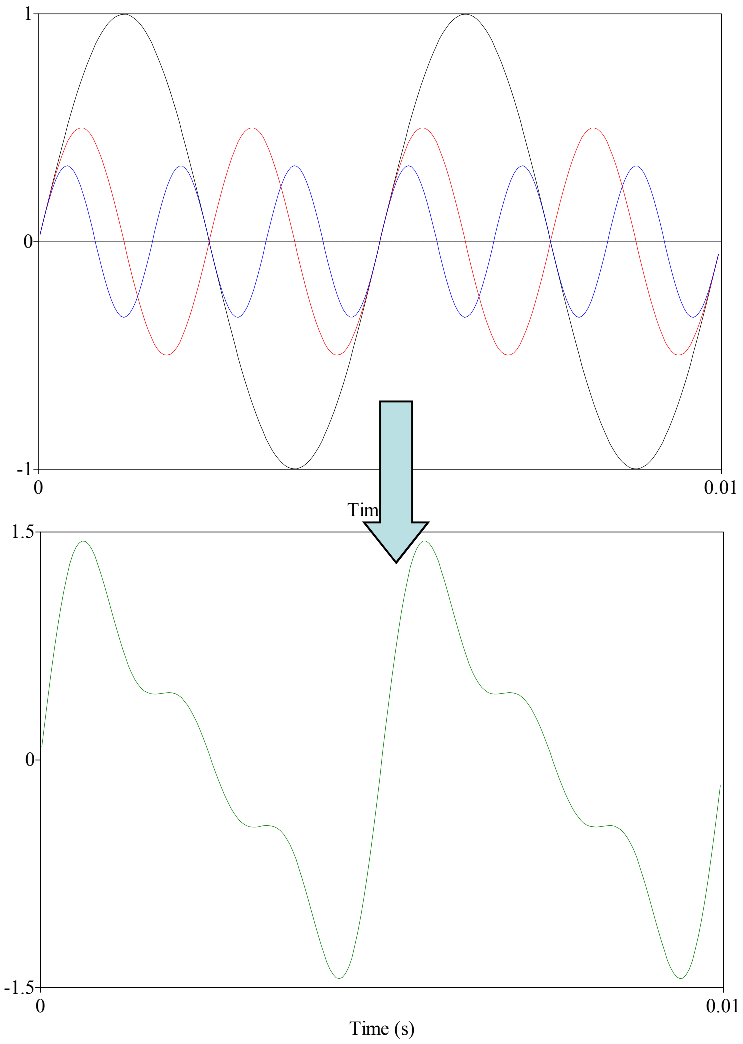
sinusoids



complex waveform

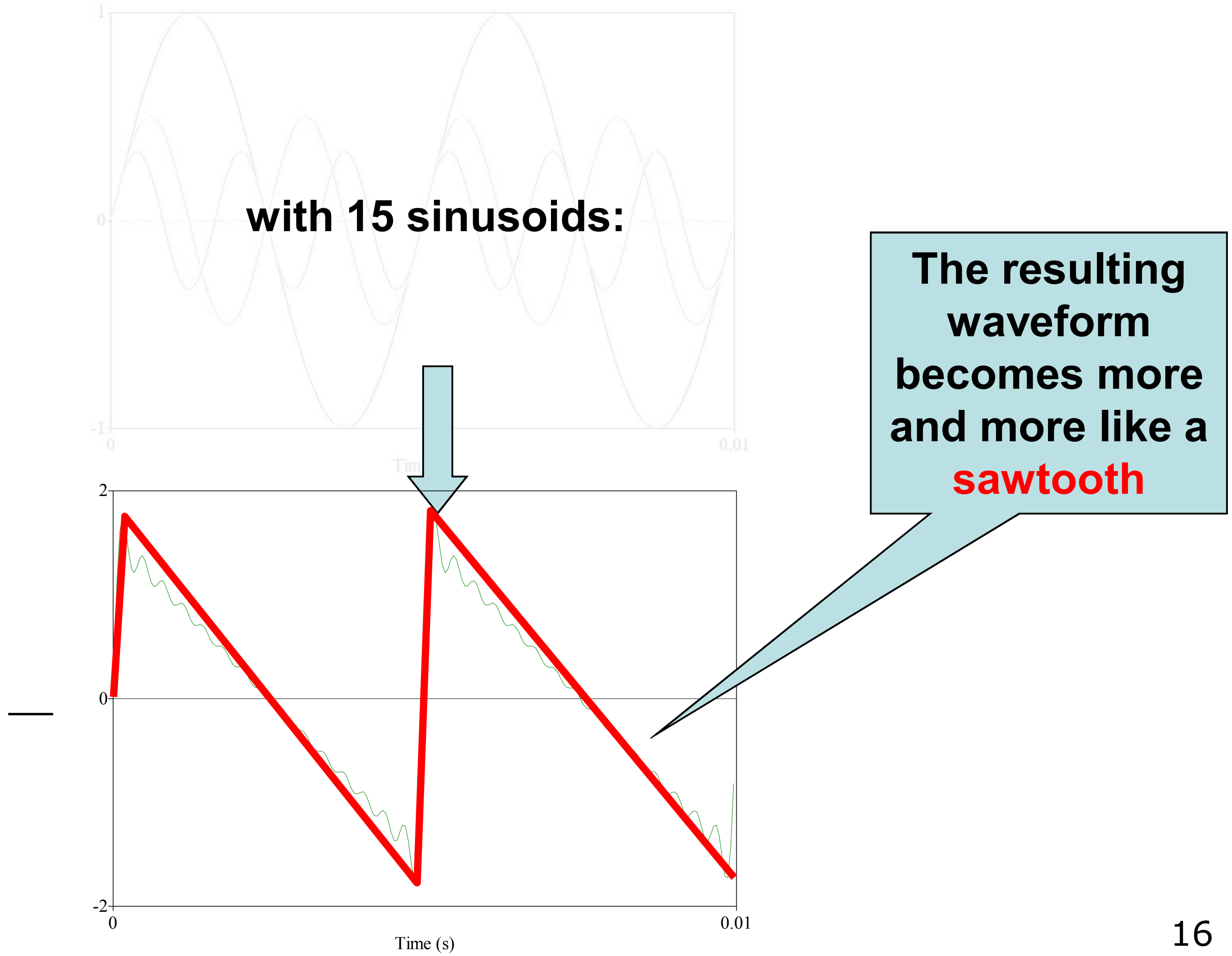


Adding a third sinusoid

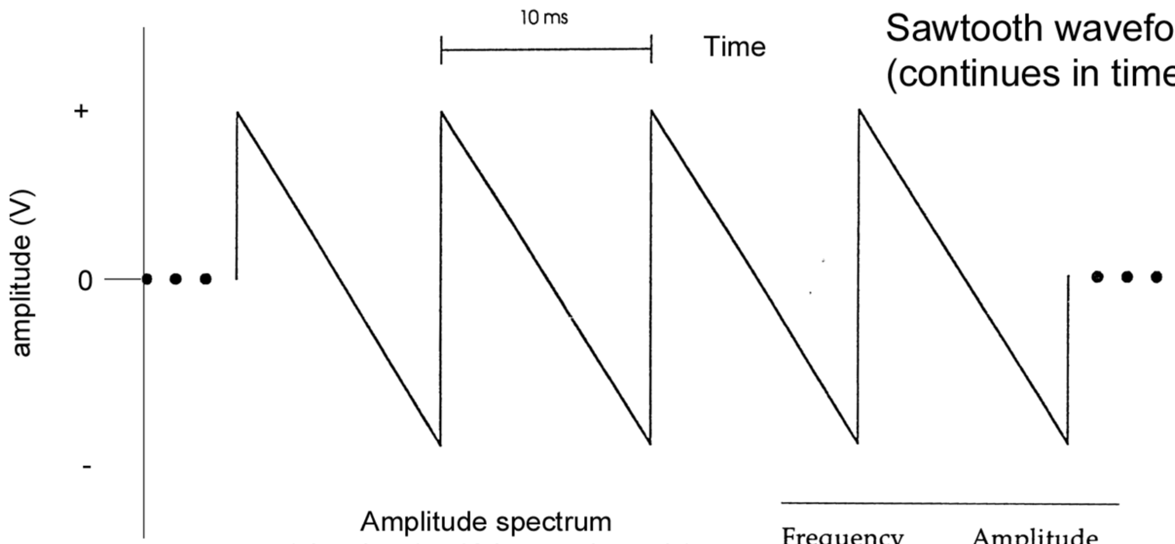


resulting periodic complex wave

Adding 15 sinusoids

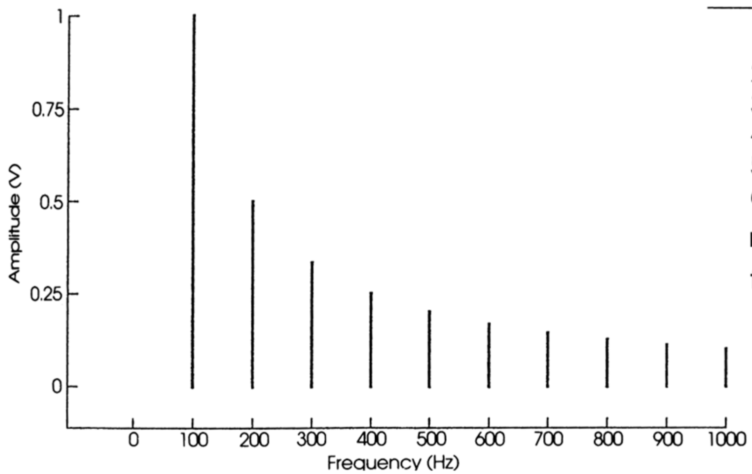


Spectrum of the sawtooth waveform



Sawtooth waveform
(continues in time...)

Amplitude spectrum
(showing 1st 10 harmonics only)



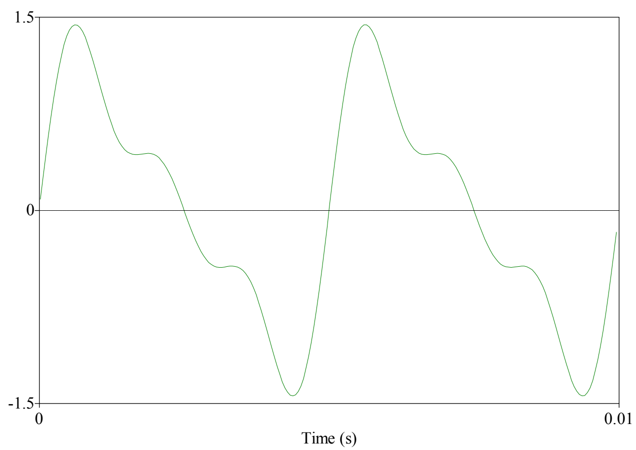
Frequency (Hz)	Amplitude (V)
100	1
200	1/2
300	1/3
400	1/4
500	1/5
600	1/6
n *x* 100	1/n

for n = 1 to infinity

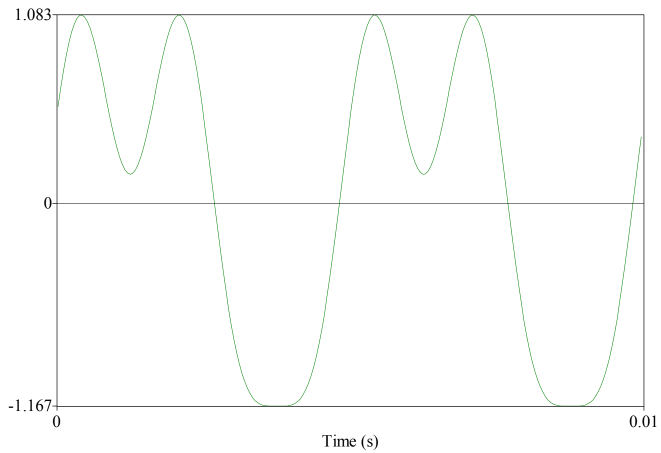
Visual effects of 'phase'

Phase can have a great effect on the resulting complex waveform, e.g.:

200, 400, and 600 Hz sinusoids added:



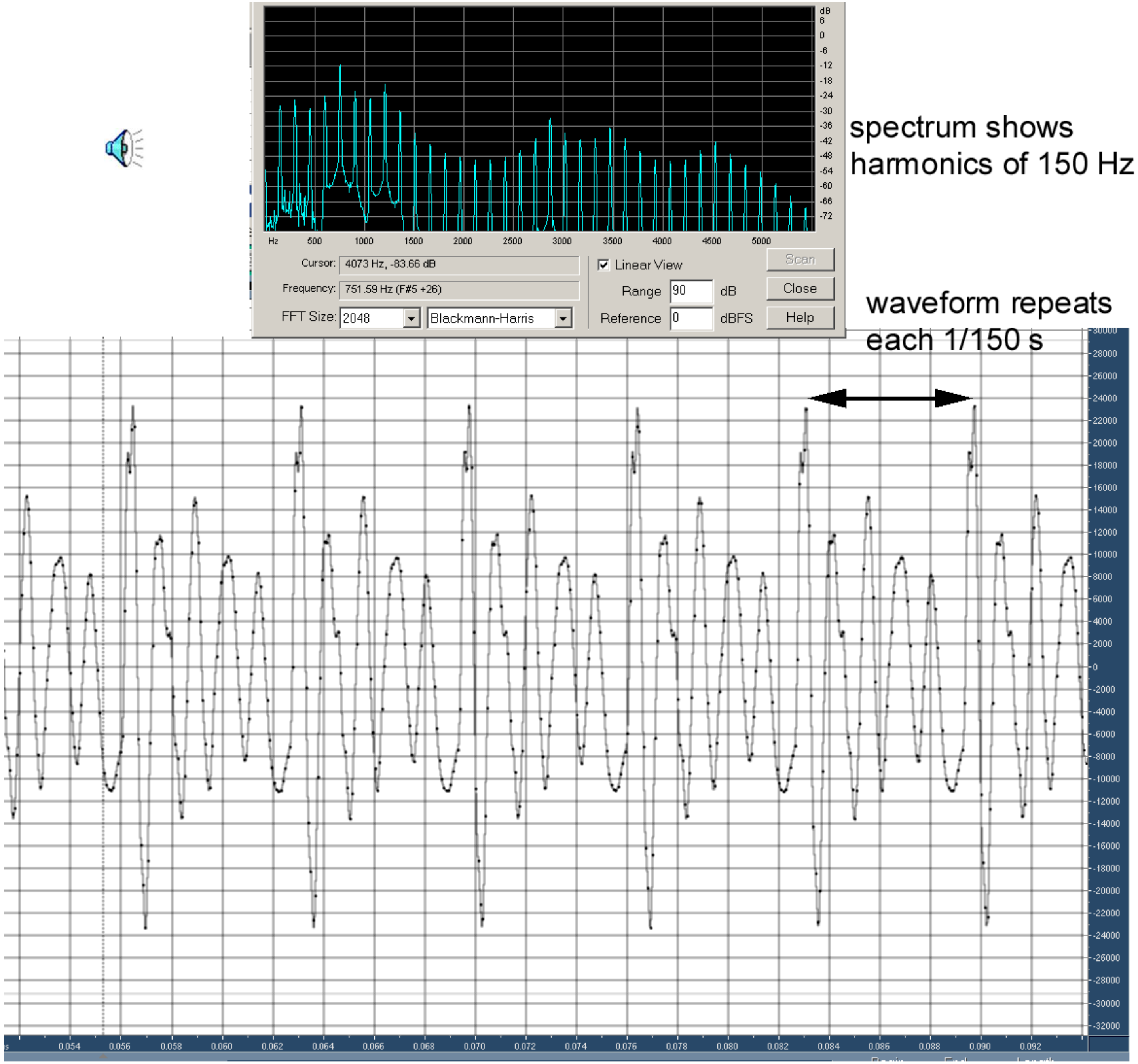
all in the same (sine) phase



400 Hz sinusoid is + 90°

Other periodic complex waves

- Infinite number of possible periodic complex wave shapes.
- *All* complex periodic waves have spectra whose sine-wave components are *harmonically-related*
  - frequencies are whole-number (integer) multiples of a common “fundamental” frequency.



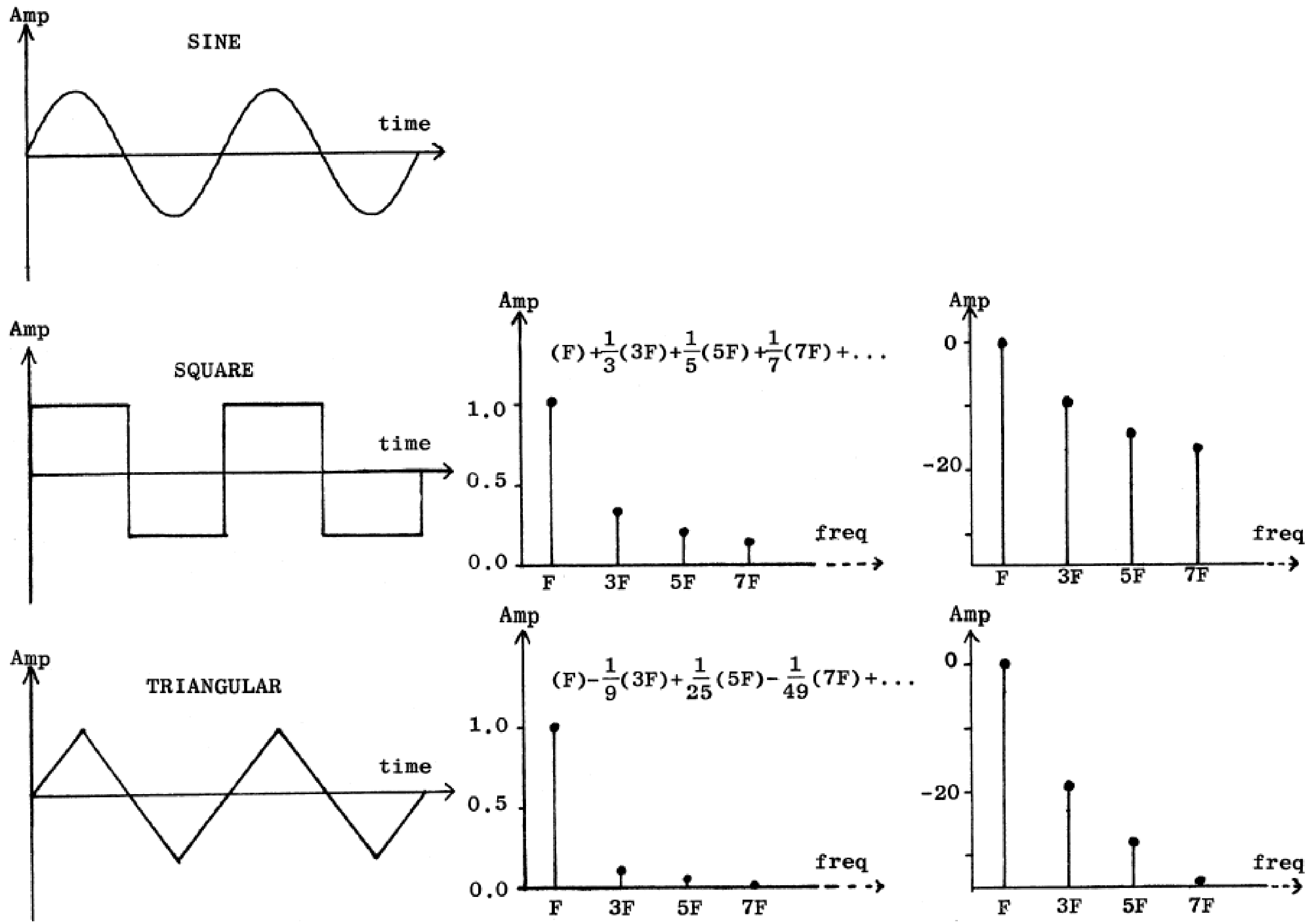
spectrum shows harmonics of 150 Hz

waveform repeats each 1/150 s

Vowel with fixed f_0

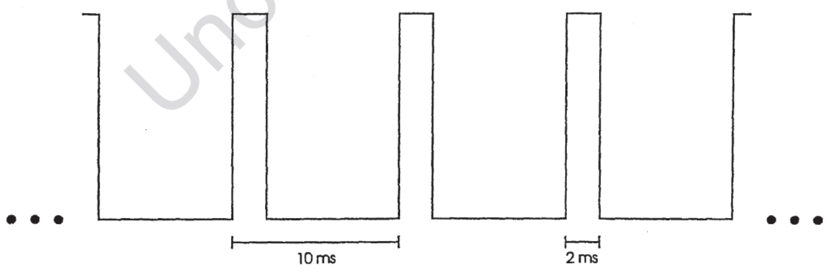
What does the spectrum of a sinusoid look like?

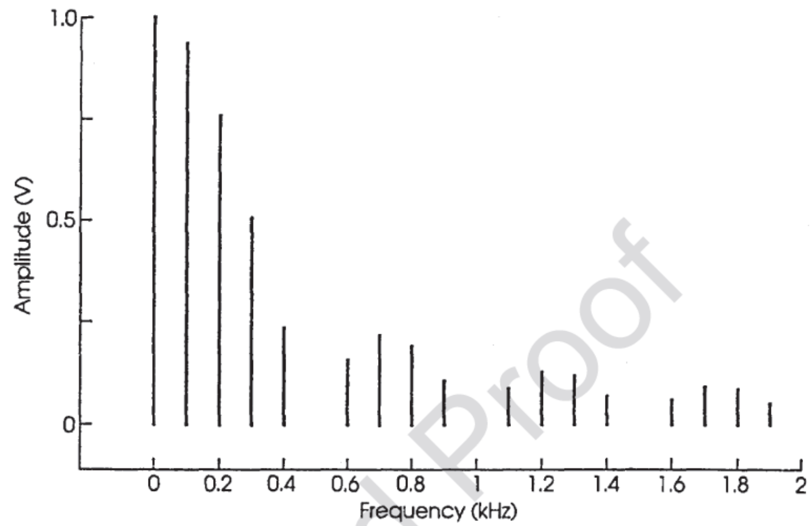
Waveform



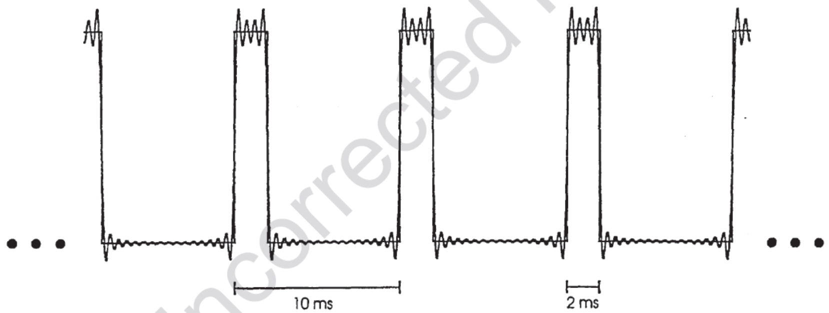
Spectrum of a pulse train

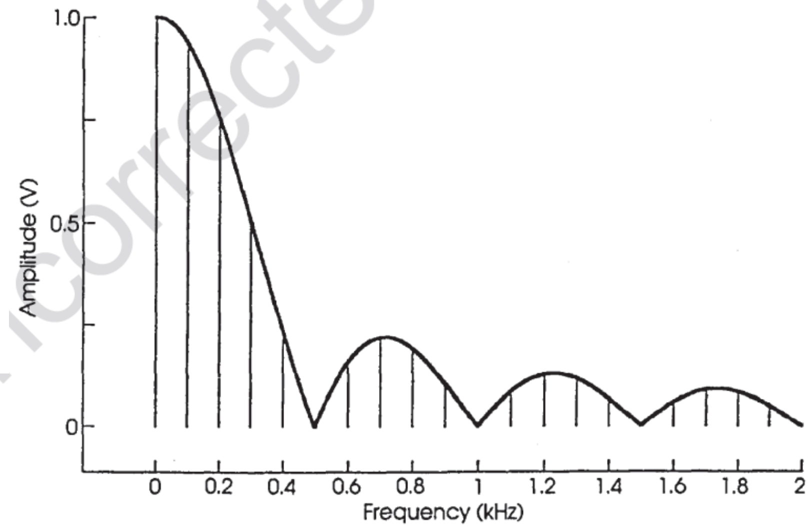
the original





the approximation



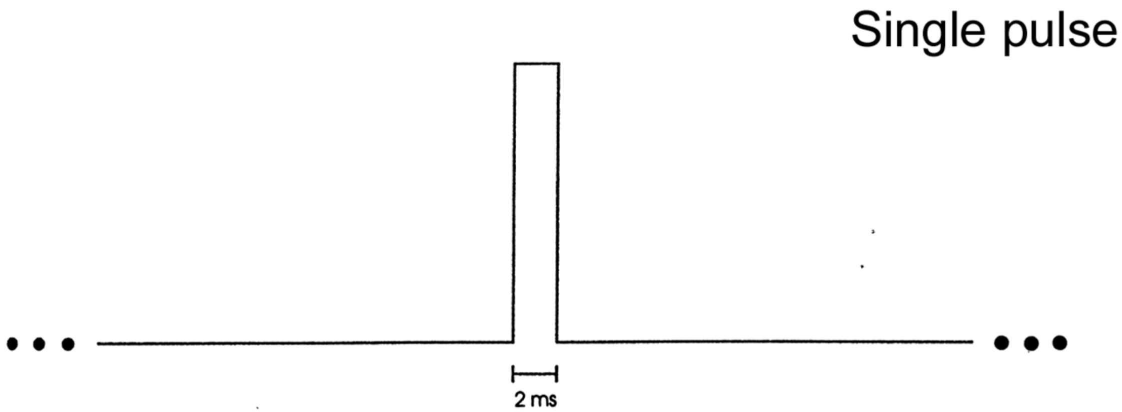


Spectra of periodic waves

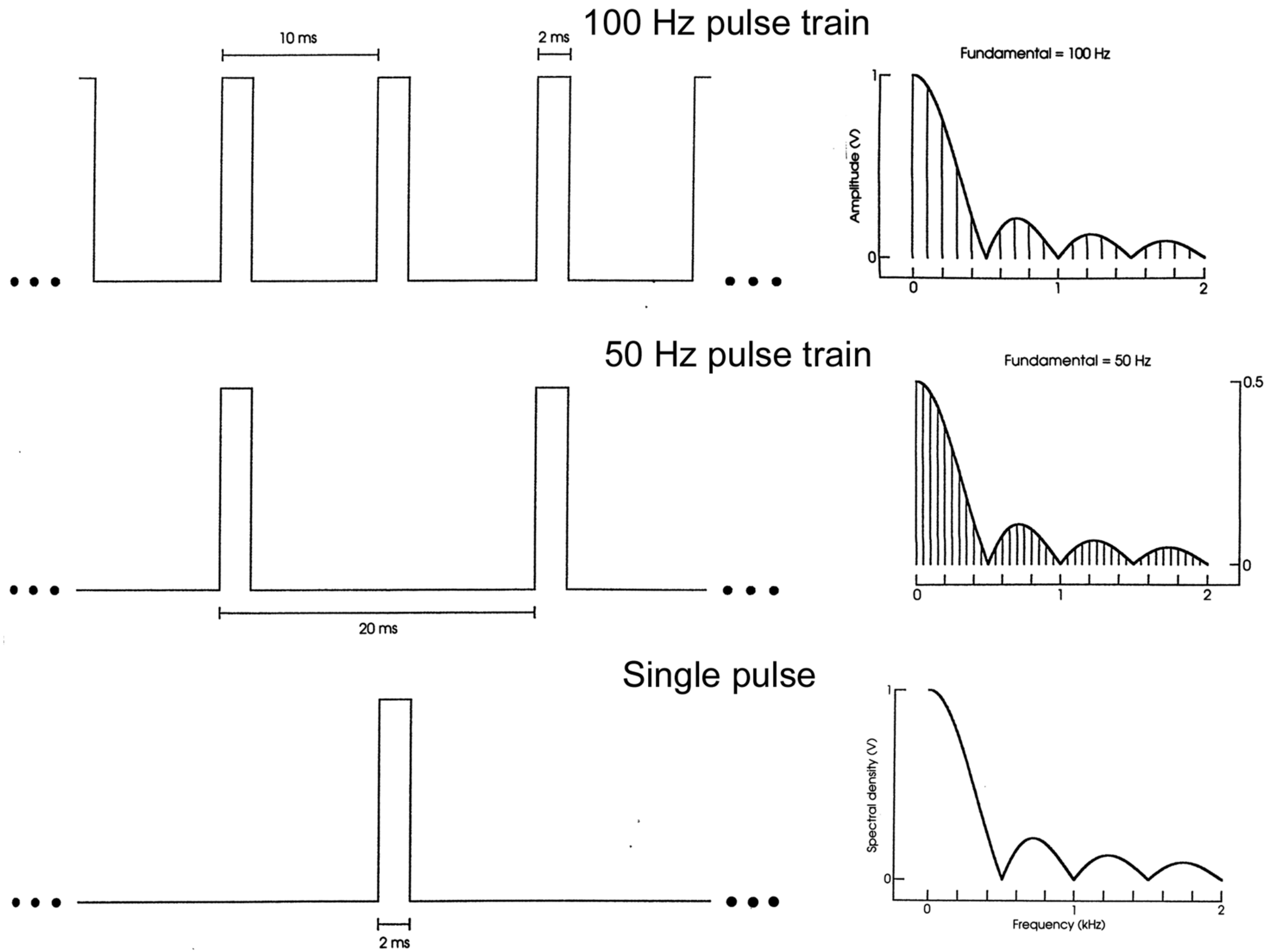
- Only the possible frequencies are constrained. The amplitude and phase of each harmonic can have any possible value
  - including zero amplitude.
- Fundamental frequency (f0) is the *greatest common factor* of harmonic frequencies.
- Series of harmonics at:
  - 100, 200, 300 Hz: f0 = 100Hz
  - 150, 200, 250 Hz: f0 = 50Hz
  - 200, 700, 1000 Hz: f0 = 100Hz

Spectra of aperiodic waves

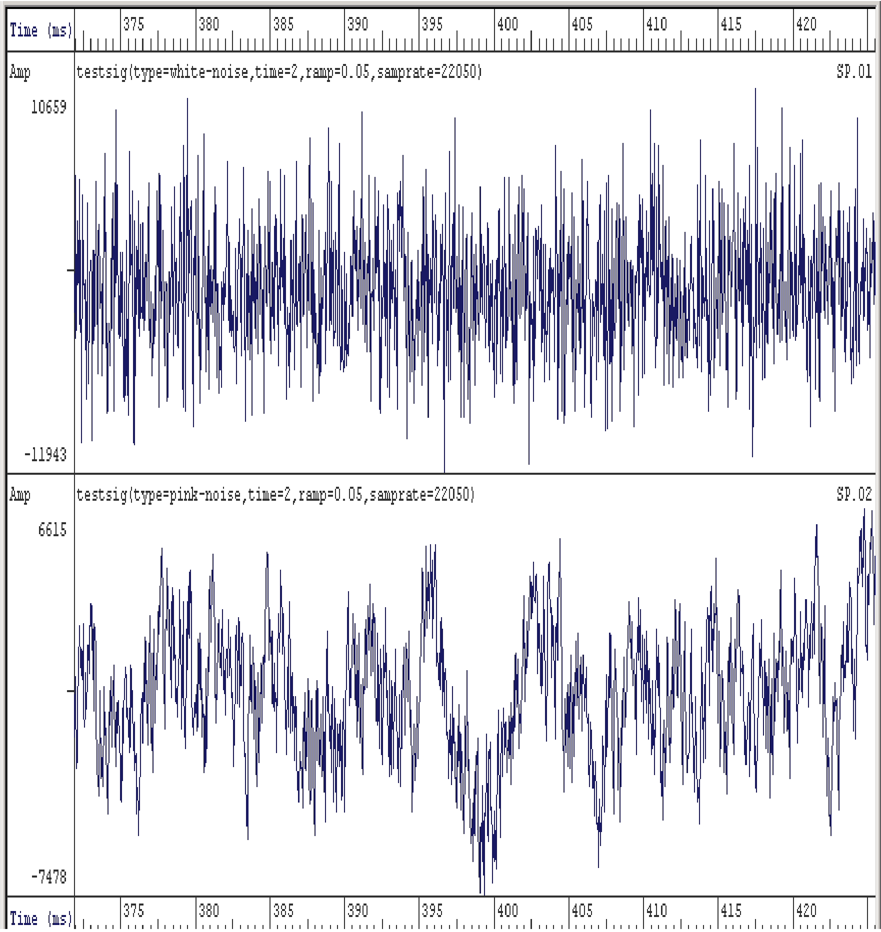
- Aperiodic waves can also be constructed from a series of sinusoids …
  - but not using harmonics only.
- Spectra are continuous – every possible frequency is present…
  - as if harmonics were infinitely close together.
- What is the spectrum of a single pulse?

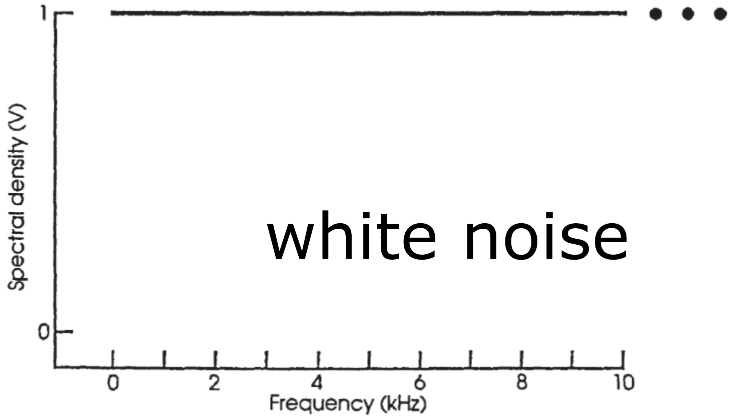


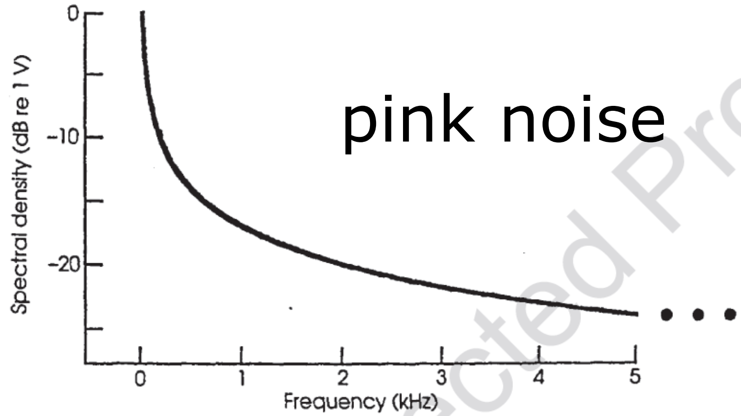
Keep lowering the fundamental frequency of a train of pulses



Spectra of random aperiodic sounds







Q: Why 'white' and 'pink'?

Q: Why 'white' and 'pink'?
A: analogies to light waves

frequency (Hz)

4×10^{14} — 7.5×10^{14}

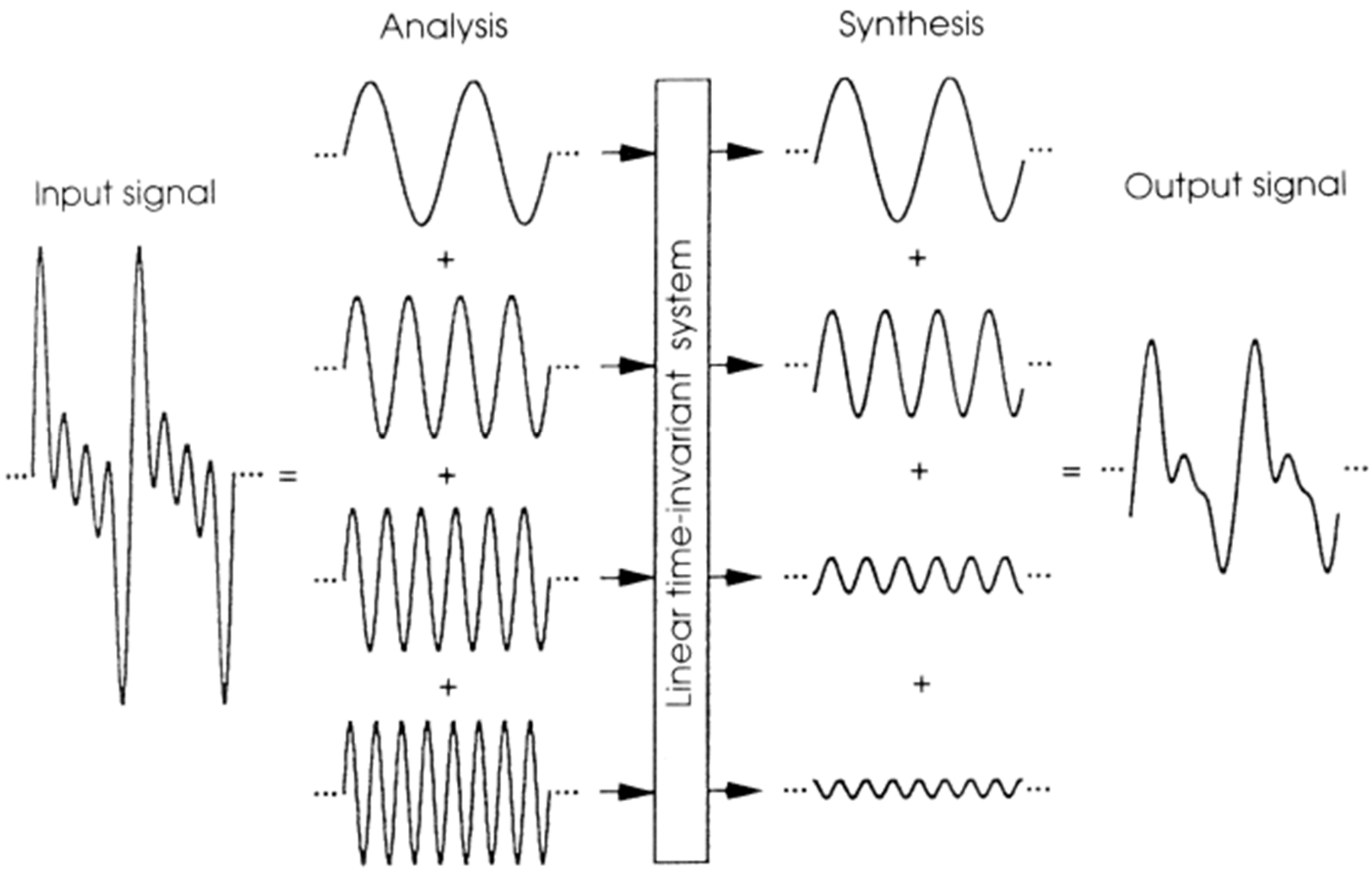
400 THz — 750 THz

kilo-	k	10^3
mega-	M	10^6
giga-	G	10^9
tera-	T	10^{12}
peta-	P	10^{15}

Key Points

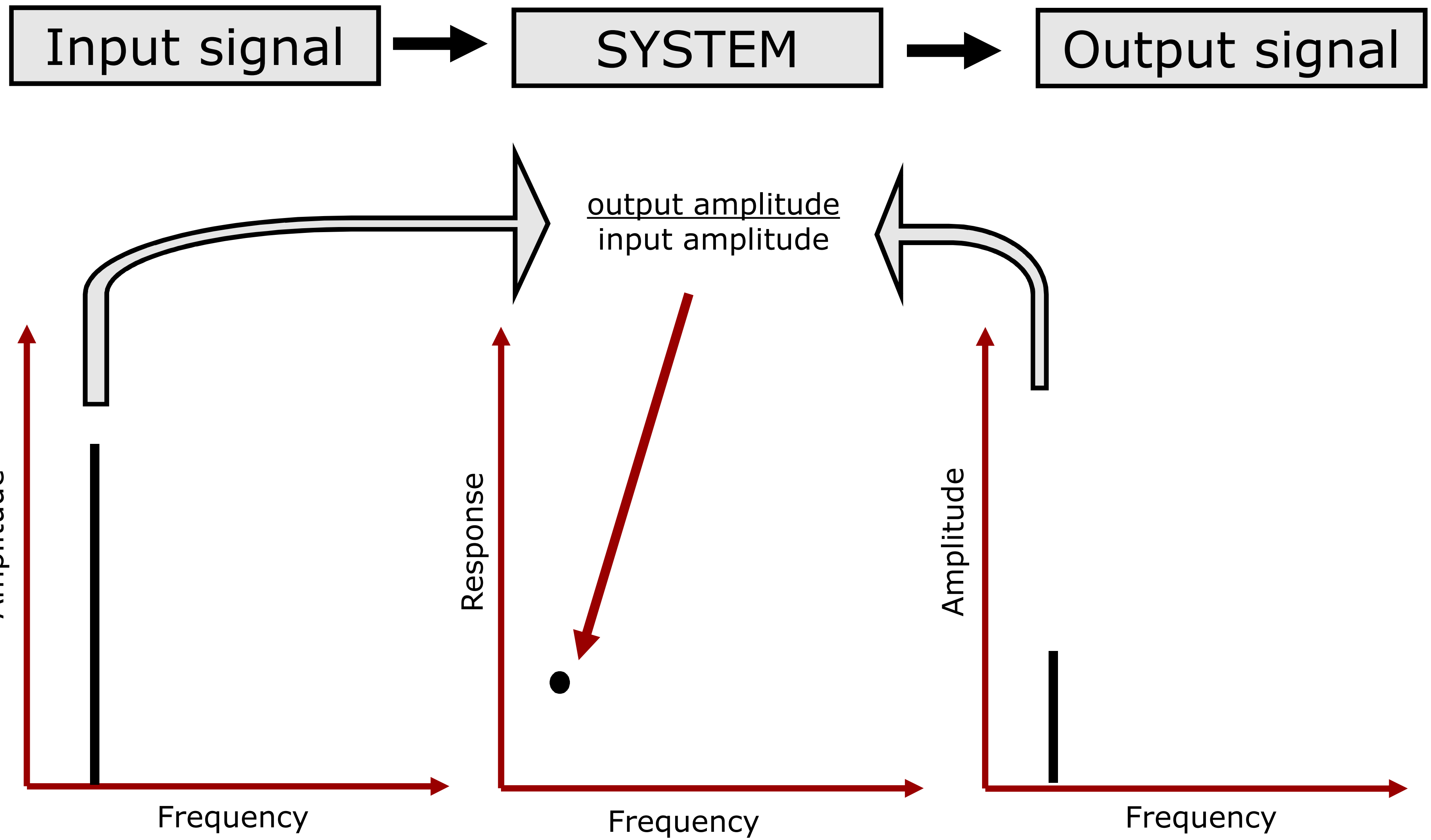
- Fourier synthesis
  - any waveform can be constructed by adding together a unique series of sine-waves, each specified by frequency, amplitude and phase …
  - but an infinite number may be needed.
- Fourier analysis
  - Any waveform can be decomposed into a unique set of component sinusoids
  - involves complex mathematics but this is easily carried out by computers and digital signal processors.
- Periodic waves have spectra that can only consist of components at harmonic frequencies of the fundamental.
- Aperiodic waves can have anything else – almost always *continuous* spectra.

The BIG idea: Illustrated

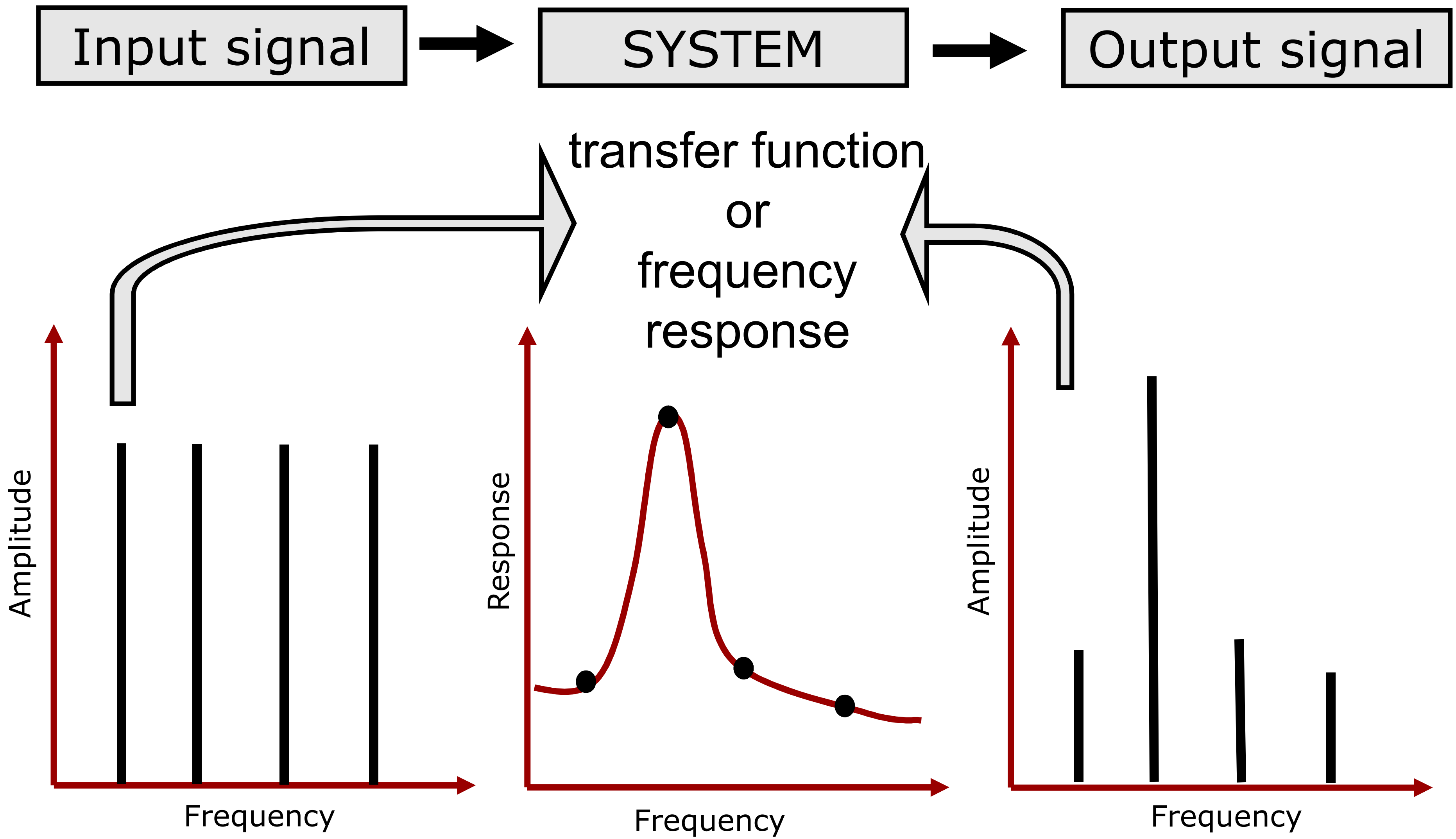


Representing systems in terms of what they do to sinusoids: Frequency responses

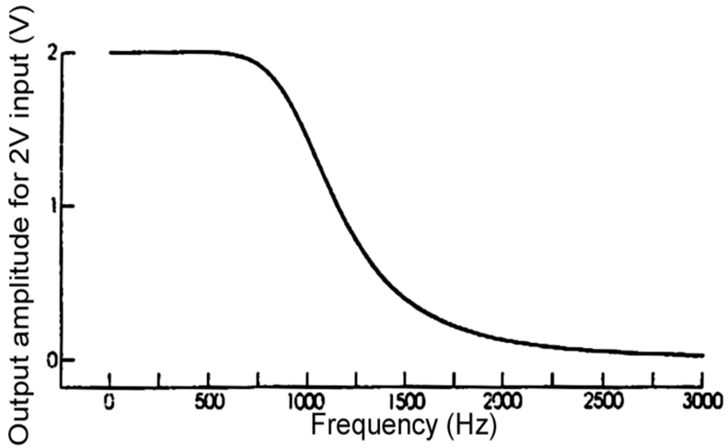
Characterisation of LTI-Systems

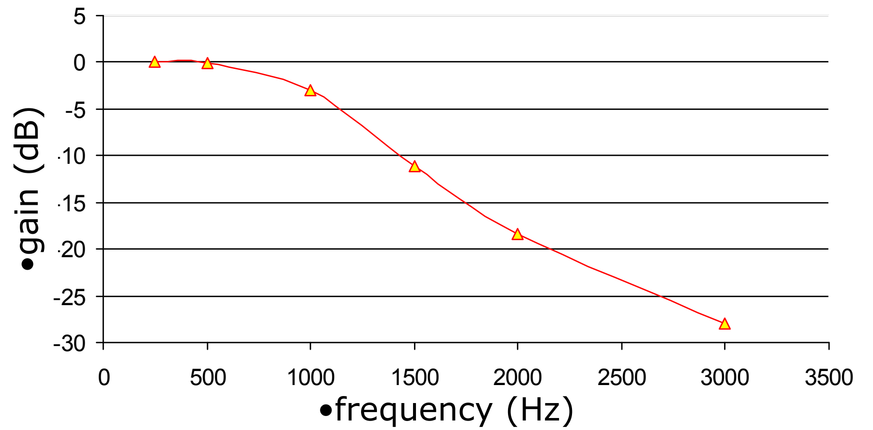


Characterisation of LTI-Systems



Amplitude Response: Key points





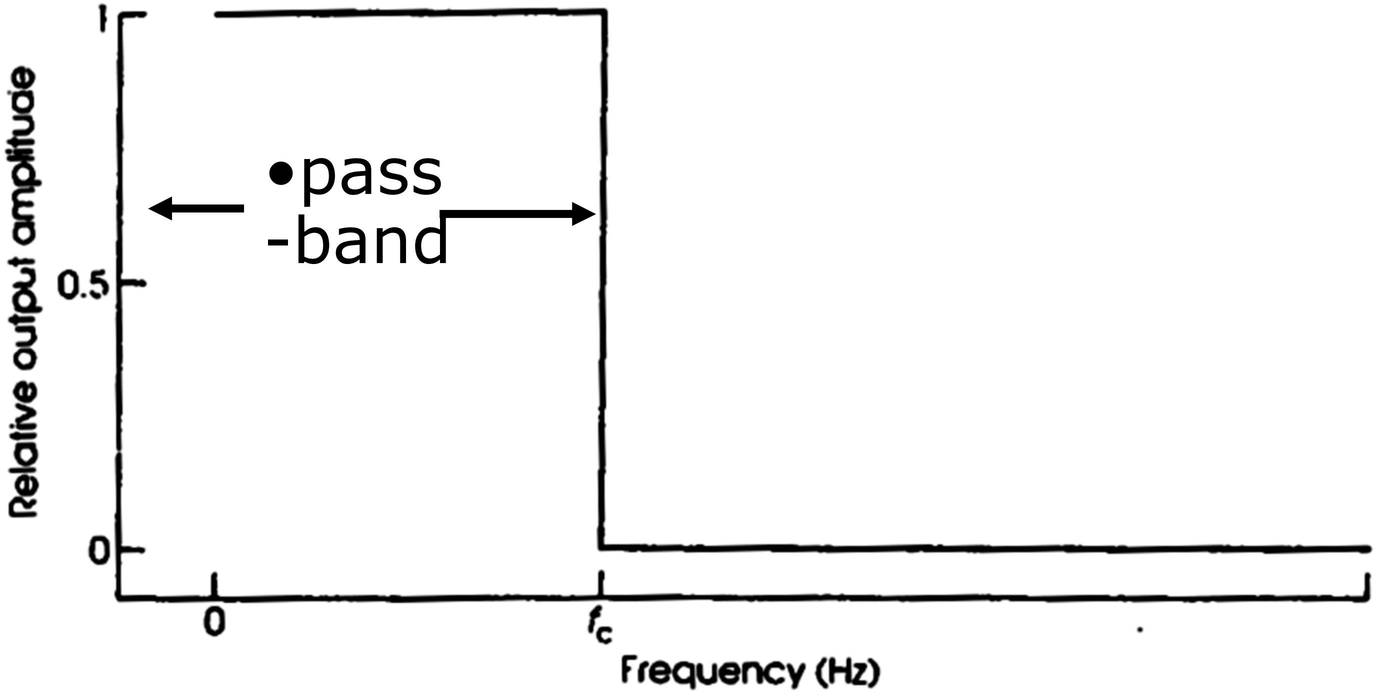
- Change made by system to amplitude of a sinewave – specified over a range of frequencies.
- Response = output amplitude/input amplitude
- Usually scaled in dB as:

  20 x log(output amplitude/input amplitude)
  = response (dB re input amplitude)

Filters

- Common name for systems that change amplitude and/or phase of waves
  - or just any LTI system
- Simple filters – low-pass and high-pass

An ideal low-pass filter

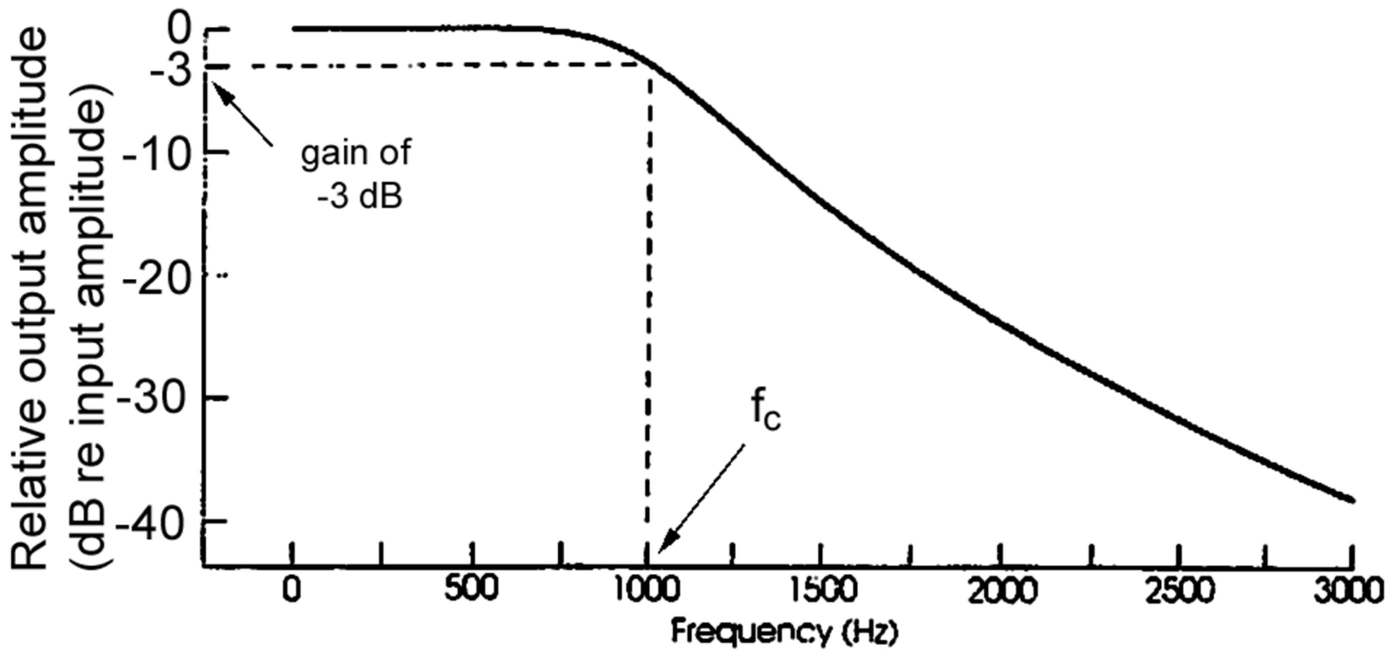


•Sudden change from gain of 1 to a very small value (virtually no output at all) at cut-off frequency f_c

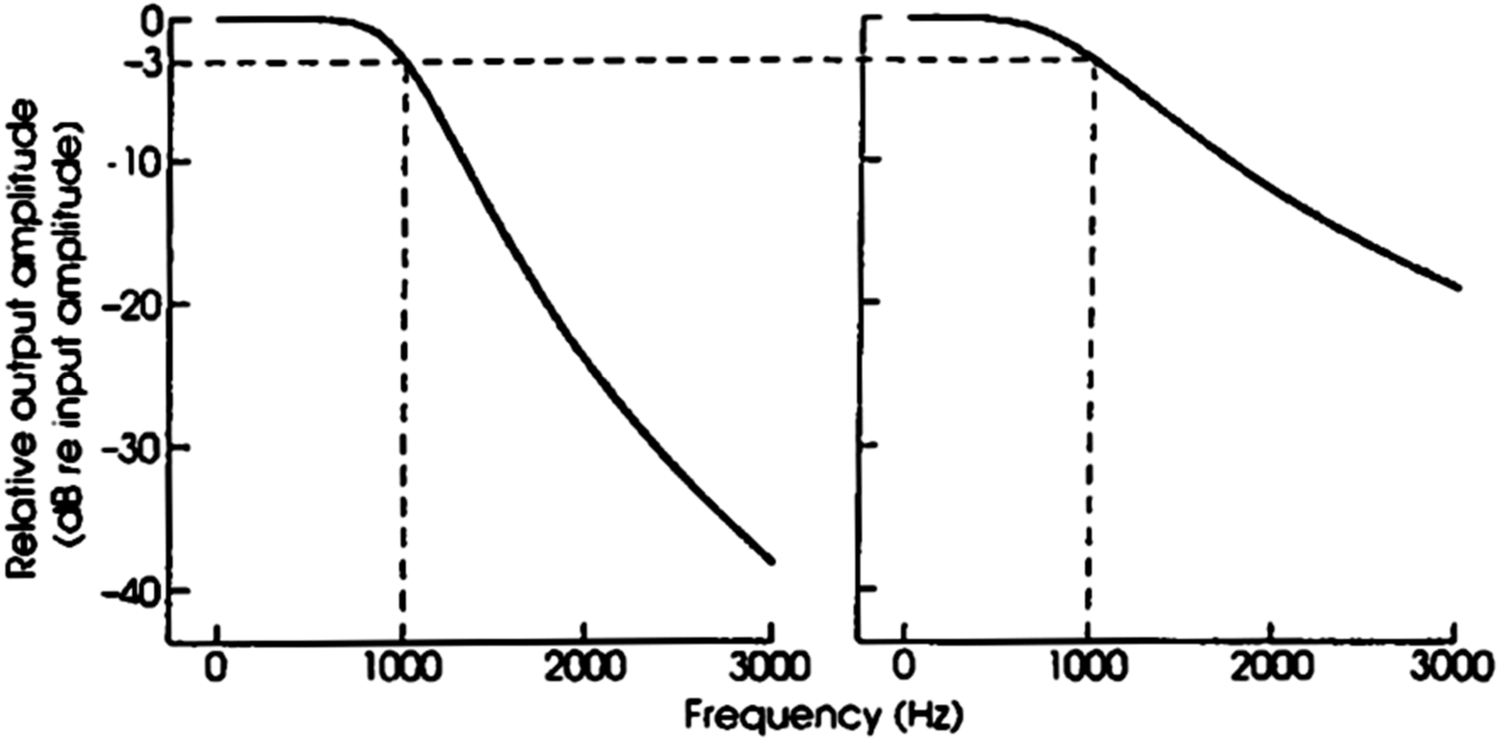
A realistic low-pass filter

- Defined as frequency where gain is -3dB.
- –3 dB is equivalent to half-*power* not half-*amplitude*

  10 log(0.5) = -3.0



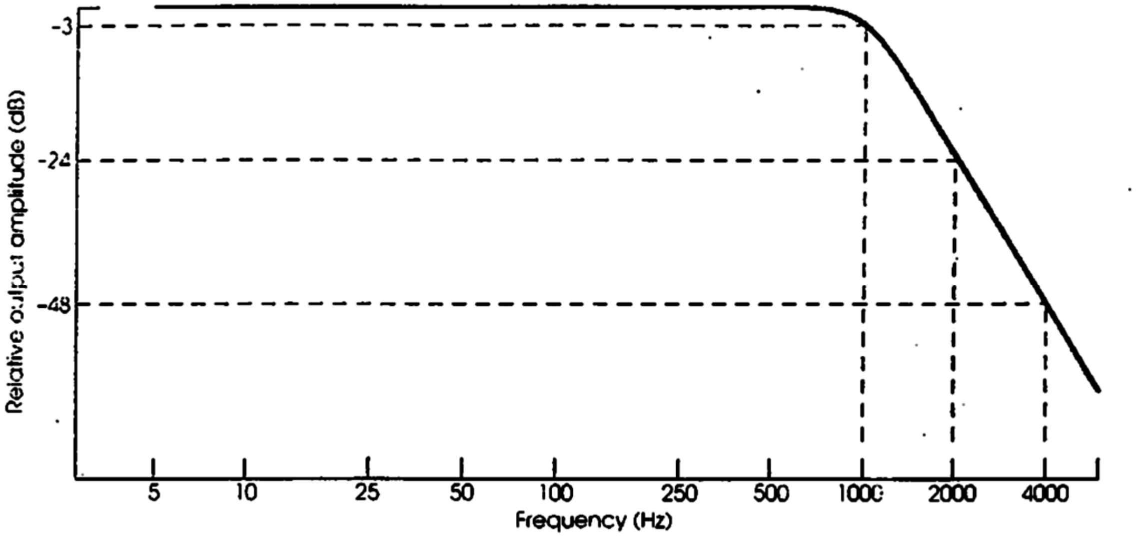
Lowpass filters can vary in the steepness of their slopes



Slope of filter

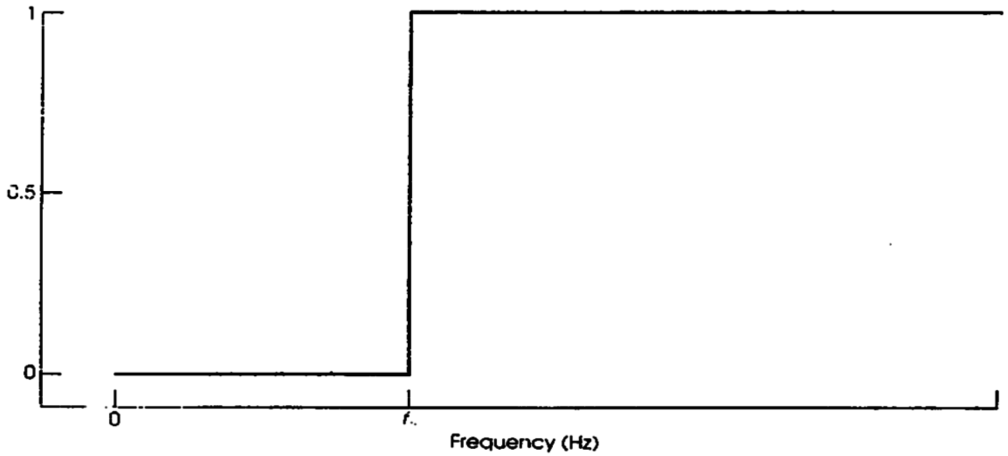
- Often constant in dB for a given frequency ratio
  - e.g., –6 dB per octave (doubling of frequency)
- suggests the use of a log frequency scale as well as a log amplitude ratio scale
    - dB in log base 10 (10, 100, 1000, etc.)
    - octave scale is log base 2, as implied in the frequency scale of an audiogram (125, 250, 500, 1000, 2000, etc).

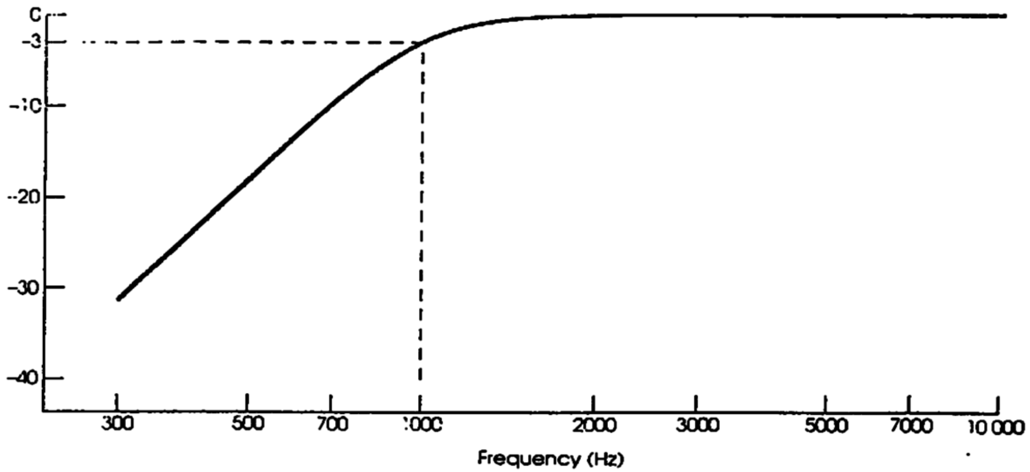
Filter slope – in dB/octave



- Degrees of steepness of slope less than18 dB/octave can be called “shallow”
- 48 dB/octave or more can be called “steep”

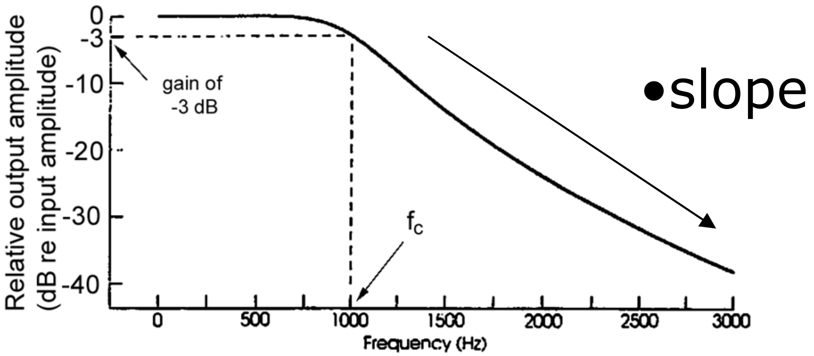
High-pass filters





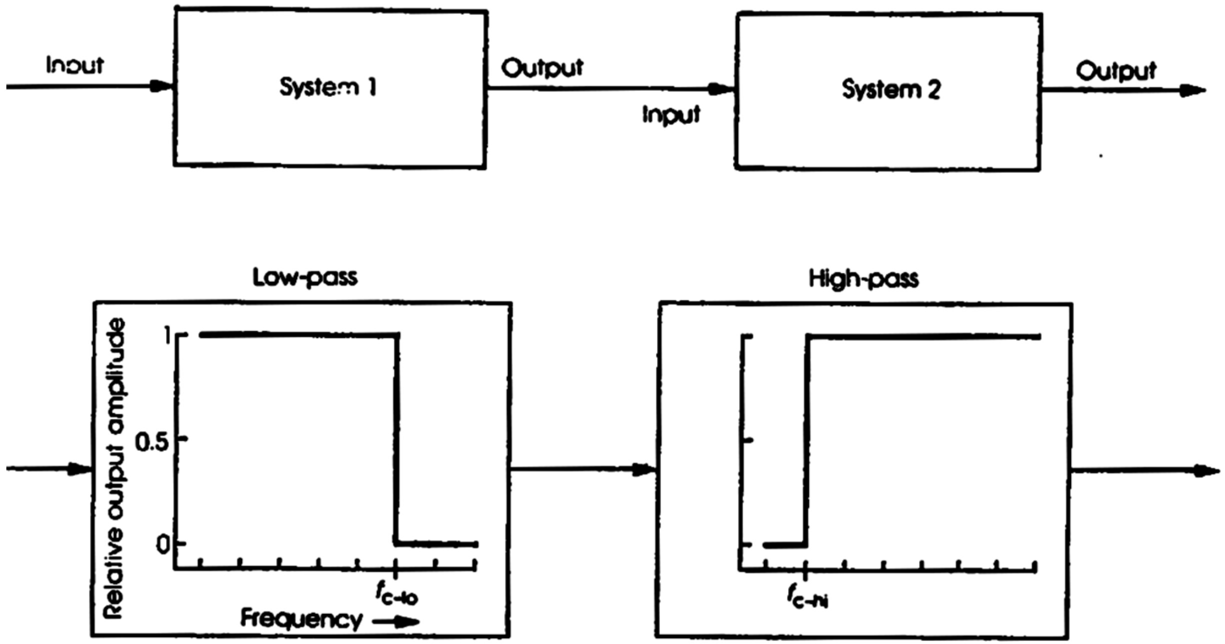
Simple filters: Key points

- High-pass or low-pass characteristics
- Defined by
  - cut-off frequency and slope of response
- Almost all natural sounds a mixture of frequencies



Systems in cascade

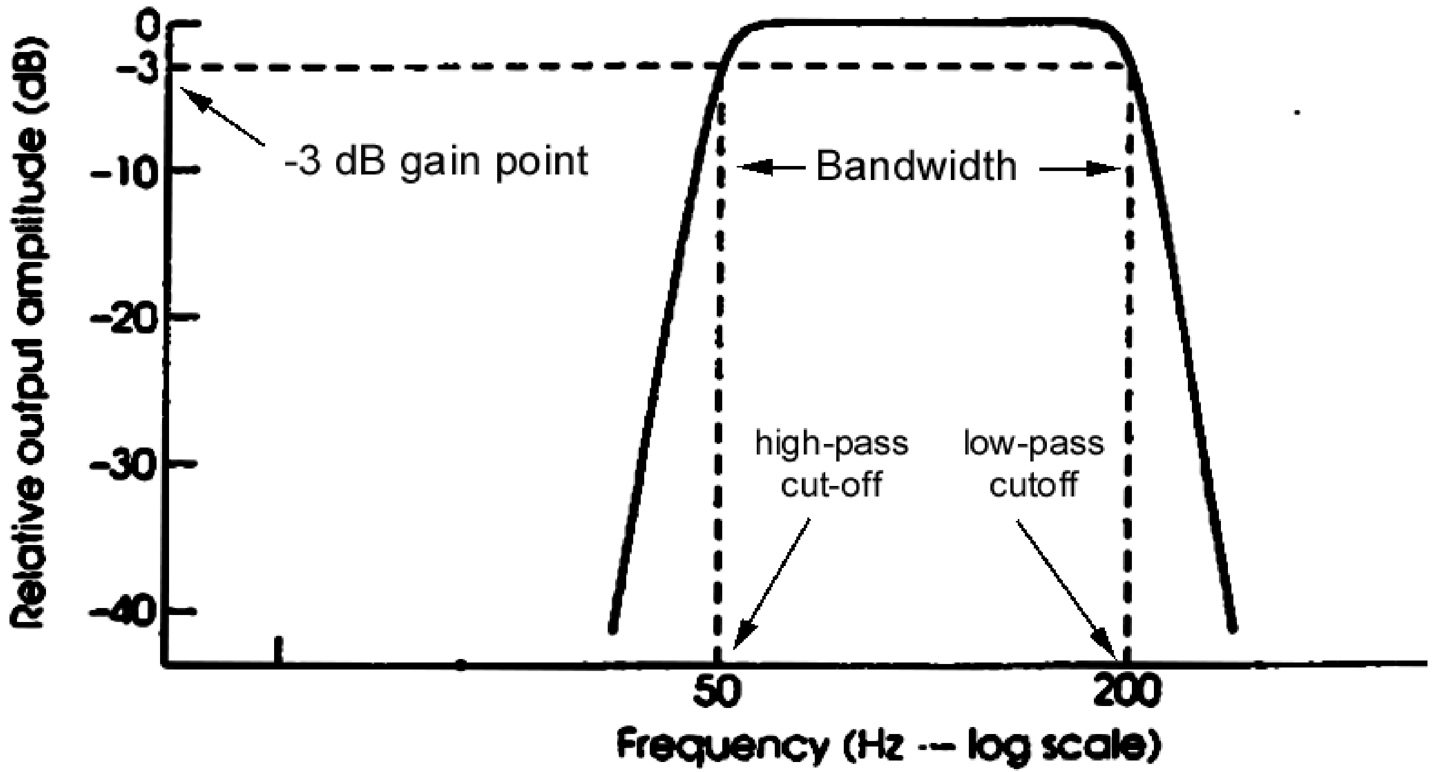
- Each stage acts independently, on the output of the previous stage



Systems in cascade

- On a linear response scale:
  - Overall amplitude response is ***product*** of component responses (*e.g.*, multiply the amplitude responses)
- On a dB (logarithmic) response scale
  - Overall amplitude response is the ***sum*** of the component responses (*i.e.*, sum the amplitude responses) …
  - Because taking logarithms turns multiplication into addition

Describing the width of a band-pass filter



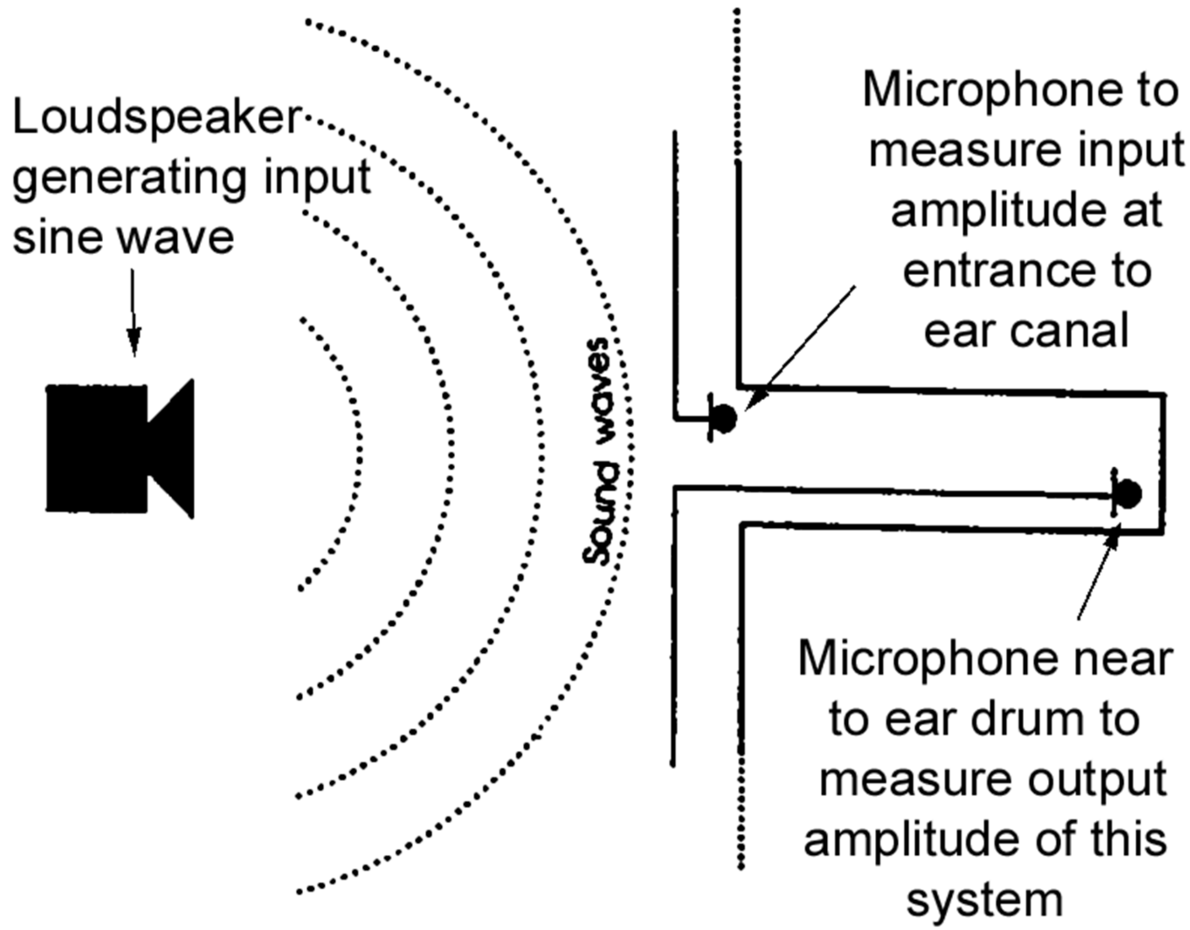
- Here bandwidth (BW) is 150 Hz

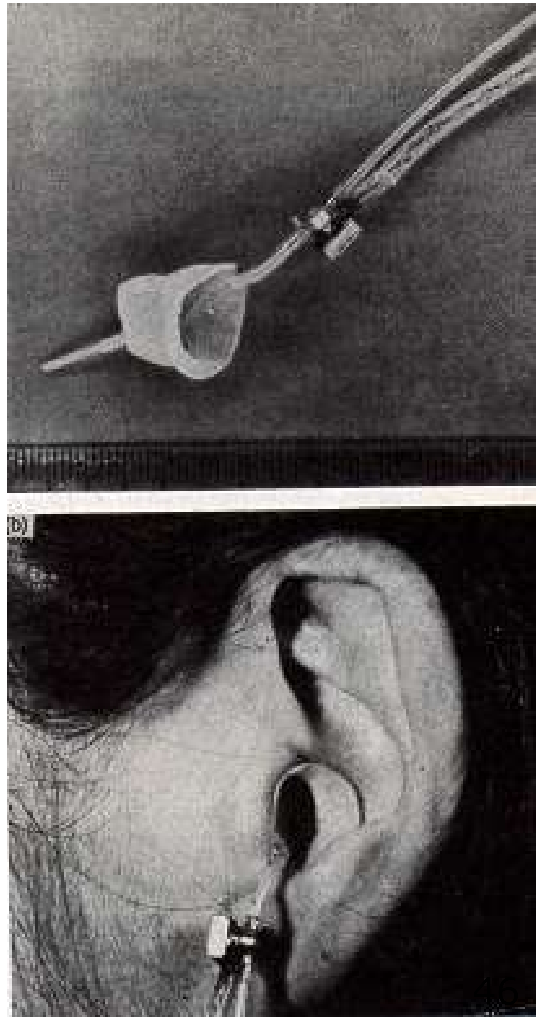
Natural filters

- Pendulum
- A relevant acoustic example:
  - a cylinder or tube closed at one end and open at the other
  - *e.g.* the ear canal

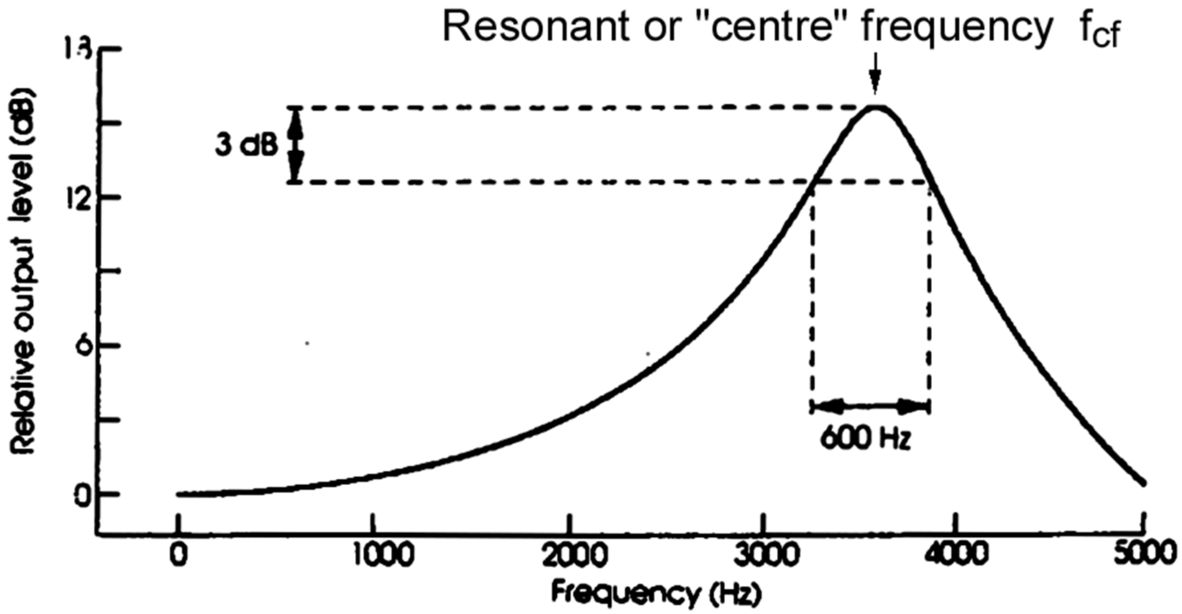
The ear canal

An acoustic tube closed at one end and open at the other (≈23 mm long)





Resonance



- Tubes like the ear canal form a special type of simple filter …
  - a resonator – similar to a band-pass filter
- Response not defined by independent high-pass and low-pass cutoff frequencies, but from a single centre frequency (the resonant frequency)
  - Resonant frequency is determined by physical characteristics of the system, often to do with size.
  - Bandwidth measured at 3 dB down points …
  - determined by the damping in the system
  - more damping=broader bandwidth

What is damping?

- The loss of energy in a vibrating system, typically due to frictional forces
- A child on a swing: feet up or brushing the floor
- A pendulum with or without a cone over the bob.
- An acoustic resonator (like the ear canal) with or without gauze over its opening
- But all systems have some damping, even if just from molecules moving against one another